MATH 410: Homework 10 **Due in class Monday 11/19/2012**

1. For each of the following f defined on $[a,b]$ first find a formula for $F(x) = \int_a^x f(t)\ dt$ for $x \in [a,b]$ which does not involve integrals. Then sketch the graph of $F(x)$.

   (a) $f : [0,10] \to \mathbb{R}$ defined by $\begin{cases} x+1 & \text{for } x \in [0,5] \\ 6 & \text{for } x \in (5,10] \end{cases}$ **

   (b) $f : [-1,1] \to \mathbb{R}$ defined by $\begin{cases} 2 & \text{for } x \in [-1,0] \\ x+1 & \text{for } x \in (0,1] \end{cases}$ **

2. One of the previous problem's $F(x)$ is not an antiderivative of the f. Identify which is not and explain why not. **

3. Suppose that $f : \mathbb{R} \to \mathbb{R}$ has continuous second derivative. Prove that **

$$f(x) = f(0) + f'(0)x + \int_0^x (x-t) f''(t)\ dt \qquad \text{for all } x$$

   Note: There's a hint somewhere in the book.

4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ has a second derivative and that

$$\begin{gathered} f''(x) - f'(x) + x f(x) = x^2 \\ f(1) = 3 \\ f'(1) = -2 \end{gathered}$$

   (a) Find and simplify $P_3(x)$ at $x_0 = 1$. *

   (b) Use your answer to (a) to approximate $f(1.1)$. Simplify. *

   (c) Use your answer to (a) to approximate $\int_{0.9}^{1.1} f$. Simplify. *

5. Prove that **

$$1 + \frac{x}{3} - \frac{x^2}{9} < (1+x)^{1/3} < 1 + \frac{x}{3} \quad \text{for } x > 0$$

6. Suppose that each of $f, g : \mathbb{R} \to \mathbb{R}$ has $n+1$ continuous derivatives. Prove that f and g have contact of order n at $x = 0$ if and only if **

$$\lim_{x \to 0} \frac{f(x) - g(x)}{x^n} = 0$$