MATH 410: Homework 12 **Due in class Wednesday 12/12/2012**

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1. For each of the following series determine convergence by finding an expression for the n^{th} partial sum and then taking the limit.

   (a) $\sum_{k=1}^{\infty} 5(0.2)^{2k-1}$ *

   (b) $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$ *

   (c) $\sum_{k=1}^{\infty} k$ *

2. Use the Cauchy Convergence Criterion for Series to show that the series $\sum_{k=1}^{\infty} (0.3)^k$ converges. **

3. For each $n \in \mathbb{N}$ and $x \in [-1, 1]$ define $f_n(x) = x^{1/(2n-1)}$. Find the function $f : [-1, 1] \to \mathbb{R}$ to which the sequence $\{f_n\}$ converges pointwise and prove that it does so. Include some sketches of some f_n and of f. Show that this convergence is not uniform. **

4. For each $n \in \mathbb{N}$ and $x \in [2, \infty)$ define $f_n(x) = \frac{1}{1+x^n}$. Find the function $f : [2, \infty) \to \mathbb{R}$ to which the sequence $\{f_n\}$ converges pointwise and prove that it does so. Include some sketches of some f_n and of f. Show that this convergence is uniform. **

5. For each $n \in \mathbb{N}$ and $x \in (-1, 1)$ define $p_n(x) = x + x(1 - x^2) + ... + x(1 - x^2)^n$. Find the function $f : (-1, 1) \to \mathbb{R}$ to which the sequence $\{p_n\}$ converges pointwise and prove that it does so. **

6. Suppose that I is a closed and bounded interval and $f : I \to \mathbb{R}$ is continuous. Prove that there is a sequence of funtions $\{f_n : I \to \mathbb{R}\}$ which converges uniformly to f. *

7. For each $n \in \mathbb{N}$ and $x \in \mathbb{R}$ define **

$$f_n(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ... + (-1)^n \frac{x^n}{n!}$$

   For each $r > 0$ prove the sequence of functions $\{f_n : [-r, r] \to \mathbb{R}\}$ is unformly convergent using the Weierstrass Uniform Convergence Criterion.

8. For each $n \in \mathbb{N}$ and $x \in (-1, 1)$ define ***

$$f_n(x) = \sqrt{x^2 + \frac{1}{n}}$$

   and define $f(x) = |x|$. Prove that $\{f_n\}$ converges uniformly to f. Prove that each f_n is continuously differentiable whereas f is not. Explain why this does not contradict Theorem 9.33 in the book.

9. For each $n \in \mathbb{N}$ and $x \in [0, 1]$ define $f_n(x) = nxe^{-nx^2}$. Prove that $\{f_n\}$ converges pointwise to the function 0 but that the sequence of integrals $\left\{\int_0^1 f_n\right\}$ does not converge to 0. Explain why this does not contradict Theorem 9.32 in the book. **