MATH 410: Homework 2 **Due in class Friday 9/14/2012**

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1. Disprove each of the following with a counterexample.
   (a) $\mathbb{Z}$ is dense in $\mathbb{R}$. *
   (b) $\mathbb{R}^+$ is dense in $\mathbb{R}$. *
2. Prove that $\mathbb{R} - \mathbb{Z}$ is dense in $\mathbb{R}$. **
3. Prove that $\mathbb{Q} - \mathbb{Z}$ is dense in $\mathbb{R}$. **
4. Using only the AP give a direct ϵ-N proof that $\lim_{n\to\infty} \frac{3}{2n+1} = 0$. *
5. Using only the AP give a direct ϵ-N verification that $\left\{\frac{2}{n^2} + \frac{1}{n} + 3\right\}$ converges to 3. **
6. By contradiction (as in class) and the ϵ-N definition show that $\left\{\frac{n}{n+1}\right\}$ does not converge to 2. **
7. Suppose that $\{a_n\}$ converges to $a < 0$. Show $\exists N$ st $n \geq N \to a_n < 0$. *
8. Define $a_1 = 1$ and for $n \geq 1$ define

$$a_{n+1} = \begin{cases} a_n + \frac{1}{n} & \text{if } a_n^2 \leq 2 \\ a_n - \frac{1}{n} & \text{if } a_n^2 > 2 \end{cases}$$

   (a) Write the first five terms of this sequence. *
   (b) Show that $\forall n$, $|a_n - \sqrt{2}| < \frac{2}{n}$. **
   (c) Use this property to show the sequence converges to $\sqrt{2}$. *
9. For the sequence $\{a_n\}$ defined in book Example 2.3 show that $\forall x \in \mathbb{Q} \cap (0, 1]$ there are infinitely many indices n such that $a_n = x$. Clarifiy what this is saying with a specific example. **