MATH 410: Homework 3 **Due in class Friday 9/21/2012**

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1. By appealing only to the definition of convergence show that if $\{a_n\}$ converges to a and $\{b_n\}$ converges to b and if $\alpha \in \mathbb{R}$ then $\{\alpha a_n + b_n\}$ converges to $\alpha a + b$. **

2. Show that $\mathbb{R} - \mathbb{Q}$ is dense by showing that every $x \in \mathbb{R}$ is the limit of a sequence of irrational numbers. **

3. Prove that the set $[2, 5] \cup \{7\}$ is closed. **

4. Justify whether each of the following sequences is monotone.

   (a) $\left\{n + \frac{(-1)^n}{n}\right\}$ *

   (b) $\left\{\frac{1}{n^2} + \frac{(-1)^n}{3^n}\right\}$ *

5. Define $\{a_n\}$ recursively by $a_1 = \sqrt{2}$ and $a_{n+1} = (\sqrt{2})^{a_n}$. Prove that $\{a_n\}$ converges. **

6. Suppose that $\{a_n\}$ is monotone. Prove that $\{a_n\}$ converges iff $\{a_n^2\}$ converges. Show that this result does not hold without the monotonicity assumption. **

7. Use book problem 5 and the Comparison Lemma to obtain another proof (not the book's) that if $|c| < 1$ then $\lim_{n\to\infty} c^n = 0$. *

8. Use book problem 5 and the Comparison Lemma to prove that $\lim_{n\to\infty} \sqrt{n}c^n = 0$. **