MATH 410: Homework 4 **Due in class Friday 9/28/2012**

1. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and $\forall x \in \mathbb{Q}, f(x) = 0$.
   (a) Prove that $\forall x \in \mathbb{R}, f(x) = 0$. **
   (b) What is important about $\mathbb{Q}$ here? State (do not prove) a generalization of the result based upon this important fact. *

2. Let $a, b \in \mathbb{R}$ with $a < b$. Find a continous function $f : (a, b) \to \mathbb{R}$ having an image that is unbounded above. Also find another continuous function $g : (a, b) \to \mathbb{R}$ having an image that is bounded above but does not attain a maximum value. Give the function rule and draw the graph for each. You do not need to prove the relevant facts about the functions. *

3. Suppose that the function $f : [0, 1] \to \mathbb{R}$ is continuous, $f(0) > 0$ and $f(1) = 0$. Prove $\exists x_0 \in (0, 1]$ such that $f(x_0) = 0$ and $f(x) > 0$ for $0 \leq x < x_0$. In other words there's a smallest $x_0 \in (0, 1]$ where $f(x_0) = 0$. **

4. Prove that there is a solution to the equation *

$$\frac{1}{\sqrt{x + x^2}} + x^2 - 2x = 0 \text{ with } x > 0$$

5. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and that $f(\mathbb{R})$ is bounded. Prove that there is a solution to the equation $f(x) = x$ with $x \in \mathbb{R}$. **