MATH 410: Homework 5 **Due in class Friday 10/5/2012**

1. For each of the following statements, determine if true or false. If false provide a counterexample. True statements need no justification.
   (a) Every continuous function $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous. *
   (b) Every continuous function $f : [0, 1) \to \mathbb{R}$ is uniformly continuous. *
   (c) Every continuous function $f : [0, 1] \to \mathbb{R}$ is uniformly continuous. *
   (d) Every uniformly continuous function $f : D \to \mathbb{R}$ is continuous. *
2. Prove that $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3$ is not uniformly continuous. *
3. Prove from the definition that $f : [1, 5] \to \mathbb{R}$ given by $f(x) = \frac{2}{x+3}$ is uniformly continuous. Don't use the theorem from class! **
4. Suppose that $f : (a, b) \to \mathbb{R}$ is uniformly continuous. prove that f is bounded. **
5. Define $f(x) = \sqrt{x}$ for $x \geq 0$. Verify the ϵ-δ criterion for continuity at $x = 4$. **
6. Define *

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq \frac{3}{4} \\ 2 & \text{if } x > \frac{3}{4} \end{cases}$$

   Use the ϵ-δ criterion to show that f is not continuous at $x = \frac{3}{4}$.
7. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \frac{1}{x^2+1}$. Prove that h satisfies the ϵ-δ criterion on $\mathbb{R}$. **