MATH 410: Homework 6 **Due in class Monday 10/15/2012**

1. Prove that the function $f : (1, \infty) \to \mathbb{R}$ given by $f(x) = \frac{x}{1-x}$ is strictly monotone. *
2. Is the converse of Theorem 3.23 true or false? Justify. *
3. Let $D = [0, 1] \cup [2, 3]$ and define $f : D \to \mathbb{R}$ by *

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ x^2 - 3 & \text{if } 2 \leq x \leq 3 \end{cases}$$

   (a) Prove that f is continuous using Theorem 3.23. **
   (b) Prove that f is continuous using the definition of continuity. **

4. Define $f : [1, 4] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2x + 1 & \text{if } 1 \leq x \leq 3 \\ x^2 & \text{if } 3 < x \leq 4 \end{cases}$$

   (a) Find $f([1, 4])$. *
   (b) Find a formula for $f^{-1} : f([1, 4]) \to [1, 4]$. **
   (c) Prove that f^{-1} is continuous. *

5. Find $\lim_{x \to 2} \frac{x-2}{x^4-16}$. *
6. Prove that $\mathbb{N}$ has no limit points. **