MATH 410: Homework 7 **Due in class Friday 10/26/2012**

1. Use the definition of the derivative to calculate $f'(x_0)$ for each of the following:
   (a) $f(x) = \sqrt{x-3}$ at $x_0 = 7$. *
   (b) $f(x) = \frac{x}{x-1}$ at $x_0 = 2$. *

2. Suppose $g : \mathbb{R} \to \mathbb{R}$ is differentiable and $f : \mathbb{R} \to \mathbb{R}$ is defined by: **

$$f(x) = \begin{cases} g(x) & \text{if } x \in \mathbb{Q} \\ -g(x) & \text{if } x \notin \mathbb{Q} \end{cases}$$

   For each x with $g(x) = 0$ show that $f'(x) = 0$ iff $g'(x) = 0$.

3. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable and there is a bounded sequence $\{x_n\}$ with $x_m \neq x_n$ whenever $m \neq n$ and $\forall n \ f(x_n) = 0$. Show $\exists x_0$ such that $f(x_0) = 0$ and $f'(x_0) = 0$. **

4. Prove or disprove: If I is a neighborhood of $x_0 \in \mathbb{R}$ and $f : I \to \mathbb{R}$ is differentiable at x_0 then f is differentiable for at least one other x-value in I. *

5. Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are each differentiable and that: **

$$\begin{aligned} &\forall x \in \mathbb{R}, f'(x) = g(x) \\ &\forall x \in \mathbb{R}, g'(x) = -f(x) \\ &f(0) = 0 \\ &g(0) = 1 \end{aligned}$$

   Prove that $\forall x \in \mathbb{R}, f(x)^2 + g(x)^2 = 1$.

6. Use the Identity Criterion to find $f(x)$ such that $f'(x) = x\sqrt{x^2+10}$ and $f(0) = 30$. *

7. Suppose that $f, g : [a,b] \to \mathbb{R}$ are continuous on $[a,b]$ and differentiable on (a,b) and satisfy $f(a) \geq g(a)$ and $\forall x \in [a,b], f'(x) \geq g'(x)$. Prove that $\forall x \in [a,b], f(x) \geq g(x)$. **

8. Deleted - Moved to next week.

9. Suppose $h : \mathbb{R} \to \mathbb{R}$ is bounded. Define $f : \mathbb{R} \to \mathbb{R}$ by **

$$f(x) = 1 + 4x + x^2 h(x)$$

   Prove that $f(0) = 1$ and $f'(0) = 4$.
   Note: You may not assume that h is differentiable.