MATH 410: Homework 8 **Due in class Friday 11/2/2012**

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1. Ascertain that $f(x) = (x-2)^5$ satisfies $f(x_0) = f'(x_0) = ... = f^{(n-1)}(x_0)$ with $x_0 = 2$ and $n = 3$. Then find all possible z satisfied by Theorem 4.24. *

2. Suppose $f : (0,4) \to \mathbb{R}$ has $f(2) = 7$, $f'(2) = 0$, $f''(2) = 0$ and $f'''(x) > \frac{1}{2}$ for $x \in (0,4)$. Find a lower bound on $f(3.9)$ and then explain why there is no upper bound. **

3. Define $f : [0,1] \to \mathbb{R}$ by $f(x) = x$.

   (a) Prove that for any partition P of $[0,1]$ we have $U(f,P) > \frac{1}{2}$. *

   (b) Prove that for each ϵ there is a partition P_ϵ of $[0,1]$ such that $U(f,P_\epsilon) \leq \frac{1}{2} + \epsilon$. **

   (c) What can you conclude about $\overline{\int_a^b} f$ from (a) and (b)? Explain. *

4. Suppose that $f : [a,b] \to \mathbb{R}$ is bounded and has $f(x) \geq 0$ for $x \in [a,b]$. Prove that $\overline{\int_a^b} f \geq 0$. *

5. Give an example of a continuous and bounded function $f : [0,2] \to \mathbb{R}$ and a sequence of partitions $\{P_n\}$ such that each P_n is composed of n subintervals, $\lim_{n\to\infty} L(f,P_n)$ exists but does not equal the area under f. Choose your f so the area is geometrically obvious. *

6. Define $f : [0,1] \to \mathbb{R}$ by **

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

   Prove that $\underline{\int_a^b} f = 0$ and $\overline{\int_a^b} f \geq \frac{1}{2}$.

7. Use the Archimedes-Riemann Theorem to show that $\int_a^b x^2 \; dx = \frac{b^3-a^3}{3}$. **

8. Define $f : [0,3] \to \mathbb{R}$ by **

$$f(x) = \begin{cases} 1 & \text{if } x \in [0,2) \\ x-1 & \text{if } x \in [2,3] \end{cases}$$

   Prove that f is integrable.