MATH 410: Homework 9 **Due in class Friday 11/9/2012**

1. Suppose that f, g, f^2, g^2 and fg are all integrable on $[a, b]$. Prove that **

$$\int_a^b fg \le \sqrt{\int_a^b f^2}\ \sqrt{\int_a^b g^2}$$

This is called the *Cauchy-Schwarz Inequality for Integrals.*
Note: There's a hint somewhere in the book.

2. Suppose that $f : [a, b] \to \mathbb{R}$ is continuous and $\int_a^b f = 0$. Prove that there is some $x_0 \in [a, b]$ such that $f(x_0) = 0$. **
Hint: Use the EVT and the IVT.

3. Suppose that $f : [0, 1] \to \mathbb{R}$ is continuous and $\forall x \in [0, 1]$ we have $f(x) \ge 0$. Prove that $\int_0^1 f > 0$ iff there is a point $x_0 \in [0, 1]$ with $f(x_0) > 0$. **

4. Show that the previous problems's claim is false if we remove the requirement that f be continous but require that it be integrable. *

5. Show by example that if the hypothesis that F be continuous on $[a, b]$ is replaced by F being defined on $[a, b]$ and continuous on (a, b) in the First Fundamental Theorem that the conclusion is false. *

6. Use the First Fundamental Theorem to evaluate $\int_0^4 x\sqrt{x^2 + 4}\ dx$. Make sure to state what F is and which requirements both F and F' satisify. *

7. Define $f : [0, 4] \to \mathbb{R}$ by **

$$f(x) = \begin{cases} 1 - x & 0 \le x \le 2 \\ x + 1 & 2 < x \le 4 \end{cases}$$

Assuming f is integrable, find $\int_0^4 f$. Justify carefully.