Theorem:
If a complete binary tree has n nodes then in the worst-case it is $\Theta(n)$ to convert it to a max heap.
Proof:
Converting to a max heap means we run `maxheapify` on the nodes with children. Those are the nodes with indices $\lfloor n/2 \rfloor$, $\lfloor n-1 \rfloor - 1$, ..., 2, 1. In the worst-case every node with children is swapped all the way to the leaf layer. We know that if the tree has n nodes then the maximum level (0-indexed) has index $\lfloor \lg n \rfloor$ but not all the leaves are necessarily at this level. However certainly the number of swaps required in total:

(a) Will be less than or equal to the number of swaps required in total if every node with children is swapped to level $\lfloor \lg n \rfloor$.

(b) Will be greater than or equal to the number of swaps required in total if every node with children is swapped to level $\lfloor \lg n \rfloor - 1$.

So let's look at these separately.

(a) Suppose every node with children is swapped to level $\lfloor \lg n \rfloor$. Here is a table showing how many nodes undergo how many swaps:

Level	# Nodes	# Swaps/Node	# Swaps
0	1	$\lfloor \lg n \rfloor$	$1(\lfloor \lg n \rfloor)$
1	2	$\lfloor \lg n \rfloor - 1$	$2(\lfloor \lg n \rfloor - 1)$
2	4	$\lfloor \lg n \rfloor - 2$	$4(\lfloor \lg n \rfloor - 2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	2^k	$\lfloor \lg n \rfloor - k$	$2^k(\lfloor \lg n \rfloor - k)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\lfloor \lg n \rfloor - 1$	$2^{\lfloor \lg n \rfloor - 1}$	1	$2^{\lfloor \lg n \rfloor - 1}(1)$
$\lfloor \lg n \rfloor$	$2^{\lfloor \lg n \rfloor}$	0	0

Since each swap takes constant time, say 1 second, then the total time equals the total number of swaps and this is then at most the sum of the rightmost column: The total number of swaps then satisfies:

$$
\begin{aligned}
T(n) &\leq \sum_{k=0}^{\lfloor \lg n \rfloor - 1} 2^k(\lfloor \lg n \rfloor - k) \\
&\leq \lfloor \lg n \rfloor \sum_{k=0}^{\lfloor \lg n \rfloor - 1} 2^k - \sum_{k=0}^{\lfloor \lg n \rfloor - 1} k2^k \\
&\leq \lfloor \lg n \rfloor \left(2^{\lfloor \lg n \rfloor} - 1\right) - \left((\lfloor \lg n \rfloor - 2)2^{\lfloor \lg n \rfloor} + 2\right) \\
&\leq \lfloor \lg n \rfloor 2^{\lfloor \lg n \rfloor} - \lfloor \lg n \rfloor - \lfloor \lg n \rfloor 2^{\lfloor \lg n \rfloor} + 2 \cdot 2^{\lfloor \lg n \rfloor} - 2 \\
&\leq 2 \cdot 2^{\lfloor \lg n \rfloor} - \lfloor \lg n \rfloor - 2 \\
&\leq 2 \cdot 2^{\lg n} \\
&\leq 2n
\end{aligned}
$$

Thus $T(n) = \mathcal{O}(n)$.

(b) Suppose every node with children is swapped to level $\lfloor \lg n \rfloor - 1$. Here is a table showing how many nodes undergo how many swaps:

Level	# Nodes	# Swaps/Node	# Swaps
0	1	$\lfloor \lg n \rfloor - 1$	$1(\lfloor \lg n \rfloor - 1)$
1	2	$\lfloor \lg n \rfloor - 2$	$2(\lfloor \lg n \rfloor - 2)$
2	4	$\lfloor \lg n \rfloor - 3$	$4(\lfloor \lg n \rfloor - 3)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	2^k	$\lfloor \lg n \rfloor - (k+1)$	$2^k(\lfloor \lg n \rfloor - (k+1))$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\lfloor \lg n \rfloor - 2$	$2^{\lfloor \lg n \rfloor - 2}$	1	$2^{\lfloor \lg n \rfloor - 2}(1)$
$\lfloor \lg n \rfloor - 1$	$2^{\lfloor \lg n \rfloor - 1}$	0	0
$\lfloor \lg n \rfloor$	$2^{\lfloor \lg n \rfloor}$	0	0

Since each swap takes constant time, say 1 second, then the total time equals the total number of swaps and this is then at most the sum of the rightmost column: The total number of swaps then satisfies:

$$\begin{aligned}
T(n) &\geq \sum_{k=0}^{\lfloor \lg n \rfloor - 2} 2^k(\lfloor \lg n \rfloor - (k+1)) \\
&\geq (\lfloor \lg n \rfloor - 1) \sum_{k=0}^{\lfloor \lg n \rfloor - 2} 2^k - \sum_{k=0}^{\lfloor \lg n \rfloor - 2} k2^k \\
&\geq (\lfloor \lg n \rfloor - 1)\left(2^{\lfloor \lg n \rfloor - 1} - 1\right) - \left((\lfloor \lg n \rfloor - 3)2^{\lfloor \lg n \rfloor - 1} + 2\right) \\
&\geq \lfloor \lg n \rfloor 2^{\lfloor \lg n \rfloor - 1} - \lfloor \lg n \rfloor - 2^{\lfloor \lg n \rfloor - 1} + 1 - \lfloor \lg n \rfloor 2^{\lfloor \lg n \rfloor - 1} + 3 \cdot 2^{\lfloor \lg n \rfloor - 1} - 2 \\
&\geq 2 \cdot 2^{\lfloor \lg n \rfloor - 1} - \lfloor \lg n \rfloor - 1 \\
&\geq 2^{\lfloor \lg n \rfloor} - \lfloor \lg n \rfloor - 1 \\
&\geq 2^{\lg(n) - 1} - \lg n - 1 \\
&\geq \frac{1}{2}n - (1 + \lg n) \\
&\geq \frac{1}{4}n \qquad \text{for large enough } n
\end{aligned}$$

Thus $T(n) = \Omega(n)$.

Together we then have $T(n) = \Theta(n)$.