**MATH 410: Exam 1 Version 1** **Spring 2019**

Directions:

- You will be graded on organization and presentation as well as correctness. Your solutions should be readable!
- Each numbered question is worth 10 points for a total of 80 points which will then be rescaled out of 100.

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1. State the following three definitions:
   (a) Define what it means for $\{x_n\} \to x_0$.
   (b) Define what it means for a set $S \subseteq \mathbb{R}$ to be closed.
   (c) Define what it means for a function $f : D \to \mathbb{R}$ to be uniformly continuous.
2. State the Intermediate Value Theorem. Pick one hypothesis, remove it, and give a counterexample showing the new statement is false.
3. The following is true for any convergent sequence $\{x_n\} \to x_0$:
$$\text{If } x_0 > 0 \text{ then } \exists N \in \mathbb{N}, \forall n \geq N, x_n > 0.$$
   State the converse and give a counterexample showing that the converse is false.
4. Prove using $\epsilon$-$N$ that:
$$\left\{2 - \frac{1}{n} + \frac{3}{n^2}\right\} \to 2$$
5. Consider $f : \mathbb{R} \to \mathbb{R}$ defined by
$$f(x) = \begin{cases} 2x & \text{if } x \leq 5 \\ 0 & \text{if } x > 5 \end{cases}$$
   Prove using the sequence definition of continuity that $f(x)$ is continuous at $x = 3$.
6. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and $x_0 \in \mathbb{R}$ with $f(x_0) > 0$. Show that there exists some $\alpha > 0$ such that $f(x) > 0$ for all $x \in (x_0 - \alpha, x_0 + \alpha)$
7. Suppose $D$ is sequentially compact and $f : D \to \mathbb{R}$ is continuous. Prove that $f(D)$ is sequentially compact.
8. Suppose $\{x_n\}$ is a bounded sequence which has the property that for all $n \in \mathbb{N}$ there is some $n_1 > n$ with $x_{n_1} > x_n$ and some $n_2 > n$ with $x_{n_2} < x_n$. Prove that $\{x_n\}$ does not converge.