MATH 410: Exam 1 Version 2 **Spring 2019**

Directions:

- You will be graded on organization and presentation as well as correctness. Your solutions should be readable!
- Each numbered question is worth 10 points for a total of 80 points which will then be rescaled out of 100.

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1. State the following three definitions:
   (a) Define what it means for $\{x_n\} \to x_0$.
   (b) Define what it means for a point $x_0 \in D \subseteq \mathbb{R}$ to be a limit point.
   (c) Define what it means for a function $f : D \to \mathbb{R}$ to be continuous.
2. State the Extreme Value Theorem. Pick one hypothesis, remove it, and give a counterexample showing the new statement is false.
3. The following is true for any monotonically increasing sequence $\{x_n\}$ and any $M \in \mathbb{R}$:

   $$\text{If } \{x_n\} \to M \text{ then } \forall n \in \mathbb{N}, x_n \leq M.$$

   State the converse and give a counterexample showing that the converse is false.
4. Prove using ϵ-N that:

   $$\left\{2 - \frac{1}{n} + \frac{3}{n^2}\right\} \to 2$$
5. Consider $f : [3, \infty) \to \mathbb{R}$ defined by $f(x) = \frac{1}{x-1}$. Prove from the definition that f is uniformly continuous.
6. Suppose $f : [a, b] \to \mathbb{R}$ is continuous and for all $\epsilon > 0$ there is some $x \in [a, b]$ with $|f(x) - 17| < \epsilon$. Prove there is some $x_0 \in [a, b]$ with $f(x_0) = 17$.
7. Problem removed due to error.
8. Suppose $\{x_n\}$ is a strictly decreasing sequence which has a convergent subsequence. Prove that $\{x_n\}$ converges.