MATH 410: Exam 2 Version 1 **Spring 2019**

- You will be graded on organization and presentation as well as correctness. Your solutions should be readable!
- Each numbered question is worth 10 points for a total of 80 points which will then be rescaled out of 100.

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1. State the following three definitions:
   (a) If I is a neighborhood of x_0, define what it means for $f : I \to \mathbb{R}$ to be differentiable at x_0.
   (b) Given $f : [a, b] \to \mathbb{R}$ and a partition $P = \{a = x_0, x_1, x_2, ..., x_n = b\}$, define the upper Darboux sum $U(f, P)$.
   (c) Define what it means for a bounded function $f : [a, b] \to \mathbb{R}$ to be integrable.
2. State the Identity Criterion. Pick one hypothesis, remove it, and give a counterexample showing the new statement is false.
3. The following is true for any continuous function $f : [a, b] \to \mathbb{R}$:

   If $\int_a^b f = 0$ then there is some $x_0 \in [a, b]$ with $f(x_0) = 0$.

   State the converse and give a counterexample showing that the converse is false.
4. Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 - 3x$. Use the definition of the derivative and the sequence definition of the limit to find $f'(-2)$.
5. Suppose $f : \mathbb{R} \to \mathbb{R}$ is such that $f(3) = 0$, $f'(3) = 1$, $f''(3) = 0$, and $f'''(x) \geq 0.02$ for all x. Use the Function Control Theorem to find a lower bound on $f(3.3)$.
6. Let $n \in \mathbb{N}$. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable and $f'(x) = 0$ has at most $n-1$ solutions. Prove that $f(x) = 0$ has at most n solutions.
7. Suppose $f : [a, b] \to \mathbb{R}$ is continuous and suppose $G : [a, b] \to \mathbb{R}$ satisfies $G(a) = 0$ and $G'(x) = f(x)$ for all $x \in (a, b)$. Prove $G(x) = \int_a^x f$ for all $x \in (a, b)$.
8. Suppose $f : [a, b] \to \mathbb{R}$ is continuous and has the property that for all c, d with $a \leq c < d \leq b$ we have $\int_c^d f \geq 0$. Prove that $f(x) \geq 0$ for all $x \in [a, b]$.