MATH 410: Exam 2 Version 2 **Spring 2019**

- You will be graded on organization and presentation as well as correctness. Your solutions should be readable!
- Each numbered question is worth 10 points for a total of 80 points which will then be rescaled out of 100.

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1. State the following three definitions:
   (a) If I is a neighborhood of x_0, define what it means for $f : I \to \mathbb{R}$ to be differentiable at x_0.
   (b) Given $f : [a, b] \to \mathbb{R}$ and a partition $P = \{a = x_0, x_1, x_2, ..., x_n = b\}$, define the lower Darboux sum $L(f, P)$.
   (c) For a bounded function $f : [a, b] \to \mathbb{R}$ define the upper Darboux integral.
2. State the Mean Value Theorem. Pick one hypothesis, remove it, and give a counterexample showing the new statement is false.
3. For an open interval I the following is true for any differentiable function $f : I \to \mathbb{R}$:

   If $f'(x) < 0$ for all $x \in I$ then f is strictly decreasing.

   State the converse and give a counterexample showing that the converse is false.
4. Define $f : [0, 3] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2 & \text{if } x \in [0,1] \\ 3 - x & \text{if } x \in (1,3] \end{cases}$$

   Write down the algebraic rule for an antiderivative of f.
5. Consider $f : [0, 2] \to \mathbb{R}$ defined by $f(x) = 2x$. You may assume f is integrable. Use the AR Theorem to calculate $\int_0^2 f$.
6. Let $x_0 \in \mathbb{R}$. Suppose $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable and satisfy $f(x_0) < g(x_0)$ and $f'(x) \leq g'(x)$ for all $x \geq x_0$. Prove that $f(x) < g(x)$ for all $x \geq x_0$.
7. Suppose $f, g : [a, b] \to \mathbb{R}$ are continuous. Prove that:

$$\int_a^b |f + g| \leq \int_a^b |f| + \int_a^b |g|$$

8. Suppose $f : [a, b] \to \mathbb{R}$ is monotone increasing. Prove that f is integrable. Of course you may not use the theorem that monotone functions are integrable!