1. Rewrite each of the following as a readable sentence following standard mathematical practice, correcting any errors and clarifying.

   (a) $x^2 - 3x - 4 = 0$, $(x-4)(x+1) = 0$, $x = 4$, $x = -1$.

   Rewrite: If $x^2 - 3x - 4 = 0$ then $(x-4)(x+1) = 0$ so that either $x = 4$ or $x = -1$.

   (b) $x = 1$, $y = 2$, $3x - 2y = -1$.

   Rewrite: If $x = 1$ and $y = 2$ then $3x - 2y = -1$.

   (c) $\exists$ a solution to $2x - 31 = 86$.

   Rewrite: There exists a solution to $2x - 31 = 86$.

   (d) If x is an integer then $2x$ is even, i.e. $x = 3$ is an integer so $2x = 2(3) = 6$ is even.

   Rewrite: If x is an integer then $2x$ is even, e.g. $x = 3$ is an integer so $2x = 2(3) = 6$ is even.

   (e) There are either 0, one or 2 solutions to a quadratic equation.

   Rewrite: There are either 0, 1 or 2 solutions to a quadratic equation.

   (f) If n is an integer then $n + m$ is also an integer.

   Rewrite: If n is an integer and m is an integer then $n + m$ is also an integer.

2. Which of the following are sets? For each which is a set give the cardinality. If not write N/A.

   (a) $1, 2, 3$

   Cardinality: N/A

   (b) $1, \{2, 3\}$

   Cardinality: N/A

   (c) $\{1, \{2, 3\}\}$

   Cardinality: 2

   (d) $\{1, 2, 3\}$

   Cardinality: 3

   (e) $\{\emptyset, 0, \{\emptyset, \{\}\}\}$

   Cardinality: 3

   (f) $\{0, 2, 4, 6, ...\}$

   Cardinality: ∞

3. Let $S = \{0, 1, 2, 3, 4, 5\}$. Describe each of the following sets as $\{x \in S \mid p(x)\}$ where $p(x)$ is some condition on x. There may be more than one way to do each so try to be as elegant as possible.

   (a) $\{0, 1, 2\}$

   Description: $\{x \in S \mid x \leq 2\}$

   (b) $\{0, 2, 4\}$

   Description: $\{x \in S \mid x \text{ is even}\}$

   (c) $\{2, 3, 5\}$

   Description: $\{x \in S \mid x \text{ is prime}\}$

4. Let $S = \{0, 1, 2, 3, 4, 5\}$. Describe each of the following sets as $\{f(x) \mid x \in S \text{ and } p(x)\}$ where $f(x)$ is a function and $p(x)$ is some condition on x. There may be more than one way to do each so try to be as elegant as possible.

   (a) $\{0, 2, 4, 6, 8, 10\}$

   Description: $\{2x \mid x \in S\}$

   (b) $\{0, 2, 4, 6\}$

   Description: $\{2x \mid x \in S \text{ and } x \leq 3\}$

   (c) $\{10, 13, 16\}$

   Description: $\{3x + 1 \mid x \in S \text{ and } x \geq 3\}$

5. Explicitly list the elements using non-conditional $\{\}$ notation in each of the following sets. For two of these you will need ellipses.

   (a) $A = \{n \in \mathbb{Z} \mid 5 < n \leq 10\}$

   Element List: $\{6, 7, 8, 9, 10\}$

   (b) $B = \{x \in \mathbb{R} \mid x^2 + 6x = -5\}$

   Element List: $\{-5, -1\}$

   (c) $C = \{x \in \mathbb{R} \mid x^2 + 3 = 0\}$

   Element List: $\emptyset$

   (d) $D = \{5x + 3 \mid x \in \mathbb{Z}\}$

   Element List: $\{..., -7, -2, 3, 8, 13, ...\}$

   (e) $D = \{4 - x \mid x \in \mathbb{Z} \text{ and } x > 7\}$

   Element List: $\{..., -7, -6, -5, -4\}$