

1. Find a collection of open intervals $S_n = (?, ?)$ enumerated by the natural numbers with

$$\bigcap_{n \in \mathbb{N}} S_n = [-1, 1]$$

Solution: We may use $S_n = (-1 - \frac{1}{n}, 1 + \frac{1}{n})$.

2. Skip.
 3. Which of the following sentences are statements? For those that are, indicate the truth value.

- (a) The number 16 is prime.

Solution: False.

- (b) Is it true that $3 \cdot 4 = 12$?

Solution: Not a statement.

- (c) $3 \cdot 4 = 12$.

Solution: True.

- (d) $\emptyset \in \emptyset$

Solution: False.

- (e) $\emptyset \subseteq \emptyset$

Solution: True.

- (f) $\emptyset \in \{\emptyset\}$

Solution: True.

- (g) $\emptyset \subseteq \{\emptyset\}$

Solution: True.

4. Suppose $p(x)$ is the open sentence $2x^2 + 5x - 3 = 0$.

- (a) Over the domain \mathbb{R} for which $x \in \mathbb{R}$ is this statement true? For which is it false? Write these in set notation.

Solution: This sentence is true for $\{\frac{1}{2}, -3\}$ and false for $\{x \in \mathbb{R} \mid x \neq \frac{1}{2} \text{ and } x \neq -3\}$.

- (b) Over the domain \mathbb{Z} for which $x \in \mathbb{Z}$ is this statement true? For which is it false? Write these in set notation.

Solution: This sentence is true for $\{-3\}$ and false for $\{x \in \mathbb{Z} \mid x \neq -3\}$.

5. Suppose $p(A)$ is the open sentence $A \not\subseteq \{1, 2\}$. For which $A \in \mathcal{P}(\{1, 2, 3\})$ is this statement true? Write this in set notation.

Solution: This is true for the set $\{\{3\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

6. Sketch the subset of $\mathbb{Z} \times \mathbb{Z}$ given by $\{(x, y) \mid x, y \in \mathbb{Z} \text{ and } 2x - y \leq 1\}$

Solution:

