

1. Negate the following statements:

(a) The number $\sqrt{57}$ is prime.

Negation:

(b) $x \in A$ and $y \in B$.

Negation:

(c) $x \in A$ or $y \notin B$.

Negation:

2. For the sets $A = \{1, 2\}$ and $B = \{1, 2, 3\}$ consider the statements

$$P : A \subseteq B \text{ and } Q : |A \cap B| = 3$$

Determine if each of the following is true or false:

Statement	T or F
P	
Q	
$P \vee Q$	
$P \wedge Q$	
$\sim((\sim P) \vee Q)$	
$P \rightarrow Q$	
$Q \rightarrow P$	
$P \leftrightarrow Q$	

3. Let $S = \{1, 2, 3, 4, 5, 6\}$ and consider the open sentences

$$P(A) : A \cap \{2, 4, 6\} = \emptyset \text{ and } Q(A) : A \neq \emptyset$$

over the domain $\mathcal{P}(S)$. Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.

Hint: What exactly does it mean to have some $A \in \mathcal{P}(S)$ with $P(A) \wedge Q(A)$ being true?

4. Suppose $P(x) : x \in [-1, 2]$ and $Q(x) : x^2 \leq 2$ over the domain $S = [-2, 2]$.

(a) For which values in the domain is the conditional $P(x) \rightarrow Q(x)$ a true statement?

(b) For which values in the domain is the conditional $Q(x) \rightarrow P(x)$ a true statement?

(c) For which values in the domain is the biconditional $P(x) \leftrightarrow Q(x)$ a true statement?