MATH 310: Groupwork 4 Solution **Thursday 7/12/2012**

1. Negate the following statements:
   (a) The number $\sqrt{57}$ is prime.
   Negation: The number $\sqrt{57}$ is not prime.
   (b) $x \in A$ and $y \in B$.
   Negation: Either $x \notin A$ or $y \notin B$.
   (c) $x \in A$ or $y \notin B$.
   Negation: Both $x \notin A$ and $y \in B$.

2. For the sets $A = \{1, 2\}$ and $B = \{1, 2, 3\}$ consider the statements

$$P : A \subseteq B \text{ and } Q : |A \cap B| = 3$$

Determine if each of the folowing is true or false:

Statement	T or F
P	T
Q	F
$P \vee Q$	T
$P \wedge Q$	F
$\sim ((\sim P) \vee Q)$	T
$P \rightarrow Q$	F
$Q \rightarrow P$	T
$P \leftrightarrow Q$	F

3. Let $S = \{1, 2, 3, 4, 5, 6\}$ and consider the open sentences

$$P(A) : A \cap \{2, 4, 6\} = \emptyset \text{ and } Q(A) : A \neq \emptyset$$

over the domain $\mathcal{P}(S)$. Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.
Hint: What exactly does it mean to have some $A \in \mathcal{P}(S)$ with $P(A) \wedge Q(A)$ being true?
Answer to hint: It means A contains none of $2, 4, 6$ but is not empty.

Thus the solution is $\{\{1\}, \{3\}, \{5\}, \{1, 3\}, \{1, 5\}, \{3, 5\}, \{1, 3, 5\}\}$.

4. Suppose $P(x) : x \in [-1, 2]$ and $Q(x) : x^2 \leq 2$ over the domain $S = [-2, 2]$.
   (a) For which values in the domain is the conditional $P(x) \rightarrow Q(x)$ a true statement?
   We need $(\sim P(x)) \vee Q(x)$ which is $x \in [-2, -1)$ or $x \in [-\sqrt{2}, \sqrt{2}]$ so the answer is $[-2, 1) \cup [-\sqrt{2}, \sqrt{2}] = [-2, \sqrt{2}]$.
   (b) For which values in the domain is the conditional $Q(x) \rightarrow P(x)$ a true statement?
   We need $(\sim Q(x)) \vee P(x)$ which is $x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, 2]$ or $x \in [-1, 2]$ so the answer is $\big([-2, -\sqrt{2}) \cup (\sqrt{2}, 2]\big) \cup [-1, 2] = [-2, -\sqrt{2}) \cup [-1, 2]$.
   (c) For which values in the domain is the biconditional $P(x) \leftrightarrow Q(x)$ a true statement?
   We need either $P(x) \wedge Q(x)$ or $(\sim P(x)) \wedge (\sim Q(x))$. The former occurs in the intersection of the answers to (a) and (b) which is $[-2, -\sqrt{2}) \cup [-1, \sqrt{2})$
   The latter occurs in the intersection of the complements of the answers to (a) and (b) which is the intersection of $(\sqrt{2}, 2]$ and $[-\sqrt{2}, -1)$ which is $\emptyset$.