1. Suppose $P(x) : x \in [-1, 2]$ and $Q(x) : x^2 \leq 2$ over the domain $S = [-2, 2]$.

2. For which values in the domain is the biconditional $P(x) \leftrightarrow Q(x)$ a true statement?
   Note: This is the final question from yesterday's groupwork.

   The solution is $[-2, -\sqrt{2}) \cup [-1, \sqrt{2}]$.

3. Suppose $P(x, y) : x^2 - y^2 = 0$ and $Q(x, y) : x = y$. Determine the truth value of $P(x, y) \leftrightarrow Q(x, y)$ for $(x, y) \in \{(1, -1), (3, 4), (5, 5)\}$.

   For $(1, -1)$ we have $P(1, -1)$ true and $Q(1, -1)$ false.
   For $(3, 4)$ we have $P(3, 4)$ false and $Q(3, 4)$ false.
   For $(5, 5)$ we have $P(5, 5)$ true and $Q(5, 5)$ true.

   Thus $P(x, y) \leftrightarrow Q(x, y)$ is true for $(x, y) \in \{(3, 4), (5, 5)\}$.

4. For statements P and Q show that $(P \wedge (P \to Q)) \to Q$ is a tautology by writing out the truth table.

P	Q	$P \to Q$	$(P \wedge (P \to Q))$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	F	T	F	T
F	T	T	F	T

5. Show that $P \to (Q \vee R) \equiv (\sim Q) \to ((\sim P) \vee R)$ by logically manipulating both sides to achieve the same statement.

   The left side is

   $$P \to (Q \vee R) \equiv (\sim P) \vee (Q \vee R)$$

   while the right side is

   $$(\sim Q) \to ((\sim P) \vee R) \equiv (\sim (\sim Q)) \vee ((\sim P) \vee R) \equiv Q \vee ((\sim P) \vee R) \equiv (\sim P) \vee (Q \vee R)$$

6. Determine with justification if the following are true or false:

   (a) $\forall n \in \mathbb{Z}$, $(2n - 1)/5 \in \mathbb{Z}$.
   False. For example for $n = 1$ we have $(2(1) - 1)/5 \notin \mathbb{Z}$.

   (b) $\exists n \in \mathbb{Z}$, $(2n - 1)/5 \in \mathbb{Z}$.
   True. For example for $n = 3$ we have $(2(3) - 1)/5 \in \mathbb{Z}$.

   (c) $\exists x \in \mathbb{Z}, \exists y \in \mathbb{R}$, $x^2 + y^2 = 3$
   True. For example for $x = 1$ and $y = \sqrt{2}$ we have $(1)^2 + (\sqrt{2})^2 = 3$.

   (d) $\sim (\exists s \in \{3, 5, 11\}, \exists t \in \{3, 5, 11\}$, $st - 2$ is not prime)
   True. By brute force we see $st - 2$ is prime for every $s \in \{3, 5, 11\}$ and $t \in \{3, 5, 11\}$ so the parenthetical part is false so the negation is true.