MATH 310: Groupwork 6 **Monday 7/16/2012**

1. Distribute the negation signs for each of the following, adjusting other symbols accordingly.

   (a) $\sim (\forall x, P(x)) \equiv$

   (b) $\sim (\exists x, Q(x)) \equiv$

   (c) $\sim (\forall x, \exists y, P(x, y) \vee Q(x, y)) \equiv$

   (d) $\sim (\exists x, \forall y, P(x, y) \wedge Q(x, y)) \equiv$

   (e) $\sim (\exists x, \forall y, P(x, y) \rightarrow Q(x, y)) \equiv$

2. Negate the following sentences, writing the results in full sentences with as few symbols as the original.

   (a) There is an $x \in \mathbb{R}$ so that for all $y \in \mathbb{Z}$ we have $x = y^2$
   Result:

   (b) For every year there is at least one day when it's sunny.
   Result:

   (c) For every week there is at least one day where it rains or snows.
   Result:

3. Each of the following is either trivially or vacuously true or both. Determine which and explain why.

   (a) Let $x \in \mathbb{R}$. Prove if $x^2 - x + 10 = 0$ then $x^3 - 7 = 0$.
   Result:

   (b) Let $x \in \mathbb{R}$. Prove if $-|x + 1| > 3$ then $x^2 \geq 0$.
   Result:

   (c) Let $n \in \mathbb{Z}$. Prove if $n^2 > 5$ then $|n + 1| \geq 0$.
   Result:

4. Provide direct proofs of each of the following:

   (a) If $x \in \mathbb{R}$ and $x^2 + x - 12 = 0$ then $x = 3$ or $x = -4$.
   Proof:

   (b) If $n \in \mathbb{Z}$ is even then $3n - 5$ is odd.
   Proof:

   (c) If $n, m \in \mathbb{Z}$ are odd then $mn^2 - n + m$ is odd.
   Proof: