MATH 310: Groupwork 7 Solutions **Tuesday 7/17/2012**

1. State the contrapositive of each of the following conditionals.
   (a) If today is Tuesday then it is raining.
   Contrapositive: If it is not raining then today is not Tuesday.
   (b) If the coffeeshop has free donuts then I'm going right now.
   Contrapositive: If I'm not going right now then the coffeeshop does not have free donuts.
   (c) If I can save up the money I'll move to either Cambodia or Laos.
   Contrapositive: If I don't move to either Cambodia or Laos then I cannot save up the money.

2. Provide proofs of each of the following by first stating and then proving the contrapositive.
   (a) Let $n \in \mathbb{Z}$. If $3n - 7$ is odd then n is even.
   Contrapositive: If n is odd then $3n - 7$ is even.
   Proof: If n is odd then $n = 2k + 1$ for some $k \in \mathbb{Z}$. Then $3n - 7 = 3(2k + 1) - 7 = 2(3k - 2) = 2b$ for $b = 3k - 2 \in \mathbb{Z}$ and so $3n - 7$ is even.
   (b) Let $x \in \mathbb{R}$. If $(x - 2)(4 - x^2) \geq 0$ then $x \leq 2$.
   Contrapositive: If $x > 2$ then $(x - 2)(4 - x^2) < 0$.
   Proof: If $x > 2$ then $x - 2 > 0$ and $4 - x^2 < 0$ so that $(x - 2)(4 - x^2) < 0$.

3. Prove by first applying a lemma (hint: you've done it) and then linking this to a direct proof.
   Let $n \in \mathbb{Z}$. If $3n - 7$ is odd then $7n + 11$ is odd.
   Proof: By a prevous problem (the lemma) we know that since $3n - 7$ is odd we have n even. Thus $n = 2k$ for some $n \in \mathbb{Z}$ and so $7n + 11 = 7(2k) + 11 = 2(7k + 5) + 1 = 2b + 1$ for $b = 7k + 5 \in \mathbb{Z}$ and so $7n + 11$ is odd.

4. Prove by first breaking up the problem into cases and then proving directly:
   If $n \in \mathbb{Z}$ then $n^3 + n$ is even.
   Proof: Case 1: If n is even then $n = 2k$ for $k \in \mathbb{Z}$ and so $n^3 + n = (2k)^3 + 2k = 2(4k^3 + k) = 2b$ for $b = 4k^3 + k \in \mathbb{Z}$ so $n^3 + n$ is even.
   Case 2: If n is odd then $n = 2k+1$ for $k \in \mathbb{Z}$ and so $n^3 + n = (2k+1)^3 + 2k+1 = 8k^3 + 12k^2 + 8k + 2 = 2(4k^3 + 6k^2 + 4k + 1) = 2b$ for $b = 4k^3 + 6k^2 + 4k + 1$ and so $n^3 + n$ is even.

5. Prove by proving the contrapositive with cases.
   Let $x, y \in \mathbb{Z}$. If xy is odd then x and y are odd.
   Contrapositive: If either x or y are even then xy is even.
   Proof: Case 1: If x is even then $x = 2k$ for $k \in \mathbb{Z}$ and so $xy = 2ky = 2(ky) = 2b$ for $b = ky \in \mathbb{Z}$ so xy is even.
   Case 2: If y is even then $y = 2k$ for $k \in \mathbb{Z}$ and so $xy = x(2k) = 2(kx) = 2b$ for $b = kx \in \mathbb{Z}$ so xy is even.
   Note that the case when both are even is actually a subset of both of the cases above since both do not depend upon the parity of the other variable.

6. Delete.