MATH 310: Homework 1 **Due Thursday 7/12/2012**

1. Rewrite each of the following as a readable sentence following standard mathematical practice, correcting any errors and clarifying.
   (a) $x^2 - 3x = 4$, $x \neq -1$, $x = 4$.
   (b) We know n is a multiple of 2, e.g. n is even.
   (c) $xy = yx$ for all x and y.

2. Which of the following are sets? For each which is a set give the cardinality. If not write N/A.
   (a) $\mathbb{Z}$
   (b) $\{\mathbb{Z}\}$
   (c) $\mathbb{Z}, \mathbb{R}$
   (d) $\emptyset$
   (e) $\{\emptyset\}$
   (f) $\{..., -4, -2, 0, 2, 4, ...\}$
   (g) $\{1, 2, \emptyset, \{2\}\}$

3. Let $S = \{0, 2, 4, 6, 8, 10\}$. Describe each of the following sets as $\{f(x) \mid x \in S \text{ and } p(x)\}$ where $f(x)$ is a function (maybe just x) and $p(x)$ is some condition (maybe no condition) on x. There may be more than one way to do each so try to be as elegant as possible.
   (a) $\{0, 1, 2, 3, 4\}$
   (b) $\{0, 4, 8\}$
   (c) $\{0, 4\}$
   (d) $\{13, 19, 25, 31\}$

4. Explicitly list the elements using non-conditional $\{\}$ notation in each of the following sets. You may or may not need ellipses.
   (a) $\{2n \mid n \in \mathbb{Z}\}$
   (b) $\{-n \mid n \in \mathbb{N}\}$
   (c) $\{5 - n/2 \mid n \in \mathbb{Z} \text{ and } n > 7\}$

5. (a) If $A = \{1, 2\}$ find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$.
   (b) If $A = \{1, 2, 3, 4\}$ find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$.
   (c) Suppose $A = \{1, 2, 3, ..., n\}$. Find (and explain how you found) a formula for $|\mathcal{P}(A)|$.

6. Find $\mathcal{P}(\emptyset)$, $\mathcal{P}(\mathcal{P}(\emptyset))$ and $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$.

7. Give examples of three sets A, B and C satisfying each of:
   (a) $A \subseteq B \subset C$
   (b) $A \in B$, $B \in C$ and $A \notin C$
   (c) $A \in B$ and $A \subset C$.

8. Let $U = \{1, 2, 3, 4, ..., 20\}$ be the universal set and let $A = \{x \mid x \in U \text{ and } x \text{ is prime}\}$ and $B = \{x \mid x \in U \text{ and } x \text{ is even}\}$. List the elements of each of the following:

   (a) $A \cup B$

   (b) $A \cap B$

   (c) $\bar{A} \cup B$

   (d) $A \cap \bar{B}$

   (e) $\overline{A \cup B}$

   (f) $\bar{A} - B$

9. Give examples of two sets A and B such that $|A - B| = |A \cap B| = |B - A| = 3$.

10. Let $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$. List the elements in each of the following sets:

    (a) $\emptyset \cap A$

    (b) $\{\emptyset\} \cap A$

    (c) $\{\emptyset, \{\emptyset\}\} \cap A$

    (d) $\emptyset \cup A$

    (e) $\{\emptyset\} \cup A$

    (f) $\{\emptyset, \{\emptyset\}\} \cup A$

11. Find a collection of closed intervals $S_n = [?, ?]$ enumerated by the natural numbers with

$$\bigcup_{n \in \mathbb{N}} S_n = (-1, 1)$$

12. Identify where each of the following series of logical steps breaks down and explain why. Your explanations may be informal but use full sentences.

    (a) If A and B are sets and $x \in A$ or $x \in B$ then $x \in A \cap B$.

    (b) If $x \in \mathbb{Z}$ and $x \not< 3$ then $n > 3$.

    (c) If $x = 1$ then $x^2 = x$ so $x^2 - 1 = x - 1$ from whence we have $(x - 1)(x + 1) = x - 1$ and so $x + 1 = 1$ giving us finally $1 + 1 = 1$.