MATH 310: Homework 2 **Due Monday 7/16/2012**

1. Categorize the following sentences as closed sentences (statements), open sentences or neither. For open sentences list (in nonconditional set notation) the elements in the domain for which the open sentence becomes a true statement.

   (a) This sentence is false.

   (b) Are there infinitely many twin primes?

   (c) There are infinitely many prime numbers.

   (d) $\frac{1}{2}$ is the square root of an integer.

   (e) $x+1$ is divisible by 3 over the domain $S = \{0, 1, 2, 3, ..., 15\}$

   (f) $A \subseteq \{1, 2\}$ over the domain $S = \mathcal{P}(\{1, 2, 3\})$.

2. Sketch the subset of $\mathbb{Z} \times \mathbb{R}$ given by

$$\left\{(x, y) \mid 2y^2 - xy \geq 0\right\}$$

3. For the sets $A = \{1, 2\}$ and $B = \mathcal{P}(\{1, 2, 3\})$ and statements $P : A \in B$ and $Q : A \cap B = B$ fill in the following table:

Statement	T or F
P	
Q	
$P \vee Q$	
$P \wedge Q$	
$\sim ((\sim P) \vee Q)$	
$P \rightarrow Q$	
$Q \rightarrow P$	
$P \leftrightarrow Q$	

4. Let $S = \{1, 2, 3, 4, 5, 6\}$ and consider the open sentences

$$P(A) : A \cup \{1, 2, 3\} = S \text{ and } Q(A) : (1 \notin A) \wedge (2 \notin A)$$

   over the domain $\mathcal{P}(S)$. Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.

5. Repeat the previous exercise with $P(A) \vee Q(A)$.

6. Suppose $P(x) : x \geq 0$ and $Q(x) : (x + 1)(x - 2) < 0$ over the domain $\mathbb{R}$.

   (a) For which values in the domain is the conditional $P(x) \rightarrow Q(x)$ a true statement?

   (b) For which values in the domain is the conditional $Q(x) \rightarrow P(x)$ a true statement?

   (c) For which values in the domain is the biconditional $P(x) \leftrightarrow Q(x)$ a true statement?