

AMSC/CMSC 460
Problem Set 1

1. a) Write a MATLAB script Mfile to calculate the values of

$$E_n = \int_0^1 x^n e^{x-1} dx$$

using the recursion relation

$$E_n = 1 - nE_{n-1}.$$

In this Mfile, start with the index $n = 1$ and go up to the index $N = 20$. Remember that $E_1 = 1/e$ and that in MATLAB this is `exp(-1)`. How can you tell that the results are incorrect? At what index n ?

b) Now write a second Mfile that uses the recursion relation starting with $N = 20$, and going backward. We know that $E_1 = e^{-1}$. Compare your computed value for E_1 with the number e^{-1} as computed by MATLAB.

2. Calculations in astrodynamics require one to evaluate the function

$$f(q) = \frac{1}{q} [1 - (1 - 2q)^{-3/2}]$$

for very small values of $q > 0$.

a) Evaluate this formula in MATLAB for $q = 10^{-8}$. What aspects of the formula for f can cause serious inaccuracies?

b) Now use the power series expansion for f at $q = 0$:

$$f(q) = -3 - \frac{3 \cdot 5}{2!} q - \frac{3 \cdot 5 \cdot 7}{3!} q^2 - \dots$$

Evaluate this formula for $q = 10^{-8}$. Compare with the result of part a). Why is this way of evaluating f more reliable?

3. Write a script Mfile to determine the unit roundoff u (machine epsilon) of the IEEE floating point number system in the form $u = 2^{-n}$. Start with $u = 1$, and then make successive divisions by 2 until MATLAB can no longer tell the difference between 1 and $1 + u$. Use the MATLAB expression `(1+u) > 1` which will produce a 1 when the inequality is true, and a 0 when it is false.

4. Problem 1.34 of Moler, page 46.

5. Problem 1.38 of Moler, page 46.

6. Problem 1.39 of Moler, page 47.

Hand in Mfiles as well as carefully edited results. Do not hand in results with error messages. You can use the diary command to combine the Mfile and the results. When possible, type your comments and explanations of results.