AMSC/CMSC 460 Problem set 4

1. Write a MATLAB programs to implement the bisection method. Input parameters should be the tolerance ϵ, call it `tol`, and the pair of points a, b which bracket the desired root x_*. Your code should take the function f as input. The output of your code should be (1) an estimate of the root, (2) an error estimate, and (3) the number of iterations.

Use your code to find the roots of the equation $e^x = 1/(.1 + x^2)$.

2. Write a MATLAB program to implement Newton's method. Input parameters should be the tolerance ϵ, and the starting point x_0. Your code should take the functions f and f' as input. You must put in a safe guard to prevent the code from doing more than 50 iterations. The output of your code should be (1) an estimate of the root, (2) an error estimate, and (3) the number of iterations.

Use your Newton code to find the positive root of the equation $x/2 = \sin x$. What happens we if take $0 < x_0 < 1$?

3. Apply your Newton code to find the zeros of $f(x) = x^3 - x^2 - x + 1$. Modify your code to print out the iterates. What is the order of convergence in each case? Why are the orders of convergence different?

4. A standard problem in quantum mechanics is to find the energy levels of a single particle in a square well potential. This leads to the problem of finding the roots of the equations:

$$\tan(s) = \frac{\sqrt{Q - s^2}}{s} \equiv p(s),$$

and

$$\tan(s) = -\frac{s}{\sqrt{Q - s^2}} \equiv q(s).$$

Set $Q = 64$. Use MATLAB to graph the curves $\tan s$, $p(s)$, $q(s)$ on the interval $[0, 8]$. From the graphs, make guesses as to the location of the roots you can see. Then write the appropriate mfiles, and use the MATLAB root finder `fzero` to find the roots with a tolerance of $\epsilon = 10^{-6}$. To see how to use `fzero`, enter `help fzero`.

5. Consider the system of equations

$$f(x, y) = xe^y - y - x^2 = 0$$

$$g(x, y) = e^{xy} - 2x - y = 0.$$

This system has two roots in the square $[0, 3] \times [0, 3]$.

To get an idea of where the roots are, plot the contour curves of both functions on the square, and look where the zero level curves intersect. Write a MATLAB program using Newton's method to find the roots with an absolute error of no more than 10^{-6}. In addition to the script mfile, you will need function mfiles for f and the Jacobian matrix

$$J(x, y) = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}.$$

To solve the linear system at each iteration you can use the MATLAB command `x = A\b` which solves the system $Ax = b$.

6. Let $f(x) = e^x - 1/(.1 + x^2)$. Use the MATLAB code `fminbnd` to find the point x_* where f attains its minimum, and evaluate $f(x_*)$.