AMSC 460/CMSC 460 Computational Methods

Problem Set 6

1. Consider the intial value problem

$$y' = -12(t-1)y, \qquad y(0) = e^{-6},$$

on the interval$[0, 5]$.

(a) Write a MATLAB program to compute approximate solutions using Euler's Method, Backward Euler Method, and Trapezoid Method with step sizes $h = 0.1,\ 0.05$, and 0.025. Because the equation is linear, you can solve for y_{n+1} in the Backward Euler method to get an explicit formula.

(b) For each method calculate the error amplification factor (as a function of t and of h). Discuss the stability of the equation and the stability of each method, indicating the dependence on t and h.

(c) Compare the exact solution $y(t) = \exp(-6(t-1)^2)$ with the computed solutions. Explain the oscillations produced by Euler's Method as well as the negative values produced by the Backward Euler Method for $h = 0.1$, and why they disappear for the smaller values of h.

(d) Finally, use the MATLAB code `ode45` on this problem on the interval $[0, 2]$. The call is `[t,y] = ode45(f, [0, 2], y0)` . Here the function $f(t, y) = -12(t-1)y$ should be given in an inline function

f = inline('-12*(t-1).*y', 't', 'y') .

y_0 is the initial value, in this case $y_0 = e^{-6}$. Compare the error at $t = 2$ for each of the computed solutions with $h = .025$ (80 steps), with the error for `ode45` at $t = 2$. How many steps does `ode45` use?

2. Problem 7.6 in the Moler text, page 221.

3. Problem 7.15 in the Moler text, page 223. For part c) use initial data $(r_0, f_0) = (90, 180)$. Try several pairs of initial data with $f_0 = 2r_0$, and r_0 approaching 100. What is happening to the periods?

4. We shall use the *shooting method* to solve the nonlinear boundary value problem

$$w''(x) = w^2(x) - 1, \quad w(0) = w(1) = 0.$$

The idea here is to solve the initial value problem with initial conditions $w(0) = 0$ and $w'(0) = \alpha$, and let α vary. In this way we define a function $\alpha \to g(\alpha) \equiv w(1, \alpha)$, and we seek that value $\alpha = \alpha_*$ such that $g(\alpha*) = 0$.

We will use `fzero` on the function g. The function mfile for g must have the parameter α as input, and the value of $w(1, \alpha)$ as the output. Here w will be computed using `ode45`. You must also make an mfile `wdot.m` for the righthand side of the differential equation, written as a system. Plot the function $w(x, \alpha_*)$.

To get a starting bracket $[\alpha_1, \alpha_2]$ for `fzero`, run the mfile `g.m` and plot the results for several values of α.