MAPL/CMSC 460

HOUR EXAM II December 6

1. Some machines do not have a reciprocal operation, i.e., they do not calculate a^{-1} directly. Instead they use a Newton routine to find a^{-1}. For a given $a > 0$, let

$$f(x) = a - 1/x.$$

a) Write out the Newton iteration scheme for $f(x) = 0$. Verify that one does not need the inverse operation even though f is defined in terms of $1/x$.

b) If $a > 0$, what is the largest interval the starting value x_0 can be in to ensure convergence?

c) How can you use the computed values x_n to estimate error $|x_n - 1/a|$?

d) Let $a = 5$. Starting with $x_0 = 1/3$, compute x_1, x_2, x_3.

2. Let $f(x)$ be a function such that $f(1) = f(0) = 0$. Newton's method is applied to find these zeros and the iterates are displayed here. What is the multiplicity of each root? Explain using the concepts of quadratic and linear convergence.

2.03867725095765	-4.153670210771533e-04
1.62788244546213	-2.769305067474566e-04
1.33680736585863	-1.846288558537228e-04
1.14498693476347	-1.230896905052619e-04
1.03991501250167	-8.206147684636669e-05
1.00412157396385	-5.470839938082788e-05
1.00005013556604	-3.647259878618027e-05
1.00000000753921	-2.431521365587297e-05
	-1.621020812729841e-05

3. If we use bisection, what precaution must be taken in finding the roots of $f(x) = \tan x - x = 0$? What can go wrong?

4. Let the simple quadrature rule on $[-1, 1]$ be given by

$$Q(f) = f(-\frac{1}{2}) + f(\frac{1}{2}).$$

a) Transform this rule to the general interval $[a, b]$.
b) What is the compound rule that can be derived from this rule?
c) Given that the simple rule has the error formula

$$\int_a^b f(x)dx = Q(f) + R$$

where $R \approx C(b-a)^3$, what is the error for the compound rule? Explain.

5. Let $f(x) = |x-1|^{1/2} \sin x$.

a) If we are using an adaptive quadrature routine, like `quad` of MATLAB, to estimate $\int_0^2 f(x)dx$, where will the greatest number of function evaluations be needed? Why?

b) How would you break up the integral, and then "soften" the singularity to make the evaluation of the integral more efficient and more accurate?