MAPL-CMSC 466
Problem set 1

Do the following problems in CK:
p52; 3, 37
p60; 3,4,5, 9b, 9e, 9h, 10.

MATLAB Problems

1. Write MATLAB programs to determine

a) unit roundoff.
b) the smallest positive number representable.

2. Graph the function $f(x) = (xe^x - 1)^3$ on the interval [.56712, .56716], in this form and in the expanded form $f(x) = x^3e^{3x} - 3x^2e^{2x} + 3xe^x - 1$. Choose the vertical axis to run from -10^{-15} to 10^{-15}. How can you explain the big difference in the graphs?

3. Write a MATLAB program that evaluates the quotient $(e^h - 1)/h$ for $h = 10^{-k}$, $k = 1, 2, \ldots, 20$ and computes the difference from the limiting value 1.

From your computation, what is the value of h that gives the best approximation to the limiting value?

4. The integrals E_n defined by

$$E_n = \int_0^1 x^n e^{x-1} dx, \quad n = 1, 2, \ldots$$

satisfy the recursion relation (for $n \geq 2$) $E_n = 1 - nE_{n-1}$ with $E_1 = e^{-1}$.

a) You can compute the E_n by hand for $n = 1, \ldots, 10$. They take the form $E_n = C_n - (-1)^n n! E_1$ for appropriate constants C_n.

b) Write a MATLAB program to evaluate the E_n for $n = 1$ to $n = 20$ using the recursion relation. How can you tell that the algorithm is unstable? What is the first index n for which the computed is obviously wrong? Compare with the exact values computed in part a) using `format long`. When do you begin to see loss of accuracy?

b) Now go to a higher value of n, say $n = 40$. Make the approximation $E_{40} \approx 1/41$. Write your program to evaluate the E_n using the recursion relation going backwards from $n = 40$ to $n = 1$. Compare your values computed this way with the exact values computed in part a) for $n = 1, \ldots, 10$, and with the values computed by the forward recursion relation in part b) for $n = 1, \ldots, 20$. Again use `format long`.