

**AMSC/CMSC 466**  
**Problem set 2**

In KC, do the following problems with pencil and paper: p 90, 6,17, 18; p106, 12.

MATLAB problems

1. Write a MATLAB program to implement the bisection method. Input parameters should be the tolerance  $\epsilon$ , call it `tol`, and the pair of points  $a, b$  which bracket the desired root  $\alpha$ . If you write your program in the form of a script Mfile, you can put the function  $f$  in the form of an inline function at the beginning of the file. The output of your code should be (1) an estimate of the root, (2) an error estimate, and (3) the number of iterations.

Use your bisection code to do problem 3, p 81, of KC.

2. Write a MATLAB program to implement Newton's method. Input parameters should be the tolerance  $\epsilon$ , and the starting point  $x_0$ . You must put in a safe guard to prevent the codes from doing more than 50 iterations. If you write your program as a script Mfile, you can put the function  $f$  as inline function at the beginning of the file. The output of your code should be (1) an estimate of the root, (2) an error estimate, and (3) the number of iterations.

Use your Newton code to do problem 5, p 92 of KC.

3. Apply your Newton code to find the zeros of  $f(x) = x^3 - x^2 - x + 1$ . Modify your code to print out the iterates. What is the order of convergence in each case? Why are the orders of convergence different?

4. A standard problem in quantum mechanics is to find the energy levels of a single particle in a square well potential. This leads to the problem of finding the roots of the equations:

$$\tan(s) = \frac{\sqrt{Q - s^2}}{s} \equiv p(s),$$

and

$$\tan(s) = -\frac{s}{\sqrt{Q - s^2}} \equiv q(s).$$

Set  $Q = 64$ . Use MATLAB to graph the curves  $\tan s$ ,  $p(s)$ ,  $q(s)$  on the interval  $[0, 8]$ . From the graphs, make guesses as to the location of the roots you can see. Then write the appropriate mfiles, and use the MATLAB root finder `fzero` to find the roots with a tolerance of  $\epsilon = 10^{-6}$ . To see how to use `fzero` and set the tolerance, enter `help fzero`.

5. Consider the system of equations

$$f(x, y) = xe^y - y - x^2 = 0$$

$$h(x, y) = e^{xy} - 2x - y = 0.$$

This system has two roots in the square  $[0, 3] \times [0, 3]$ .

To get an idea of where the roots are, plot the contour curves of both functions on the square, and look where the zero level curves intersect. Write a MATLAB program using Newton's method to find the roots with an absolute error of no more than  $10^{-6}$ . In the script Mfile, you will need inline functions for  $f$ ,  $f_x$ ,  $f_y$ ,  $h$ ,  $h_x$ , and  $h_y$ . To solve the linear system at each iteration you can use the MATLAB command `x = A\b` which solves the system  $Ax = b$ .