AMSC/CMSC 466 Problem set 4

In KC do problems p326, 27; p 327, 34; p336, 3; p348, 1.

MATLAB problems

1. Using pencil and paper, find the unique polynomial of degree ≤ 3 that interpolates the data

$$\begin{array}{ccccc} x & 0 & 1 & 1.5 & 2 \\ y & -1 & 3 & 2 & 4 \end{array}.$$

(a) First find the form that uses the Lagrange basis functions.

(b) Next use the Newton form with divided differences. Verify that both methods yield the same cubic polynomial.

2. (a) Write a MATLAB program that finds the unique polynomial of degree $\leq n$ that interpolates the data points $(x_0, y_0), \ldots, (x_n, y_n)$, using divided differences. Input should be two vectors, $\mathbf{x} = (x_0, \ldots, y_n)$ and $\mathbf{y} = (y_0, \ldots, y_n)$. Output should be the vector of coefficients $\mathbf{c} = (c_0, \ldots, c_n)$. Check your code with the example of problem 1.

(b) Write a function mfile which takes the coefficients $\mathbf{c}$ and the vector of ordinates $\mathbf{x}$ and evaluates the polynomial

$$p(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + \ldots + c_n(x - x_0) \ldots (x - x_{n-1})$$

using the extended version of Horner's method.

(c) Let $f(x) = 1/(1 + x^2)$ be the Runge example. Use your code of part (a) to interpolate f on $[-5, 5]$ with equally spaced points $-5 = x_0 < x_1 < \ldots < x_n = 5$. Use $n = 5, 10, 20$. Then plot the polynomials together with the graph of f using your code of part (b). Look at the plots and comment on what happens as n increases. Where does the polynomial fit well, and where does it fit poorly?

(d) Repeat the questions of part (c) with $f(x) = |x|$ on $[-5, 5]$.

3. (a) Using pencil and paper, find the natural cubic spline for the data points

$$\begin{array}{cccc} x & 0 & 1 & 2 \\ y & 1 & 2 & 0 \end{array}.$$

Plot the spline on $[0, 2]$.

(b) Find the complete spline with the additional data $y_0' = 1.5,\ y_2' = -1$. Plot the spline on $[0, 2]$.

(c) Use the MATLAB routine `spline` with the data of part (a). Plot the result on $[0, 2]$. The MATLAB spline routine does the "not a knot" spline. Compare the three plots to see the difference.

4. Use the MATLAB spline routine to approximate the Runge function on $[-5, 5]$ with the same equally spaced points as in problem 2 for $n = 5, 10, 20$. Compare the results with the high degree polynomial interpolation.

5. Let $\mathbf{x} = (0, 1, \ldots, 8)$. Let $\mathbf{y} = (1, 1, 1, d, 1, 1, 1, 1, 1)$ where d is to be varied.

(a) Use your code of problem 2 to find the polynomial of degree 8 which interpolates these points for values of d close to 1, say, $d = 1.5, 2$. Plot the curves. Note how the whole curve is affected by a small perturbation at one point.

(b) Use the MATLAB spline routine to find the cubic spline which fits the same data. Compare the graphs of the spline with that of the interpolating polynomial. Which is less sensitive to the perturbation at a single data point? Which would be better to interpolate experimental data?