MATH 241
MATLAB Problem Set 3

This problem set is based on material in chapters 9 and 10 in the *MATLAB Companion*. You will need to download the MATLAB Mfile `simp2.m` from the web. It is contained in the Mfiles for the MATLAB Companion. The web address is `www.math.umd.edu/~jec`.

How to use `simp2`. First write an inline function for $f(x,y)$ and then pick a rectangle $R = [a,b] \times [b,c]$ over which you want to integrate R. Set `corners = [a,b, c,d]`. Then the call for `simp2` is `simp2(f, corners)`. After you make the call, the program pauses to let you enter the number n of subdivisions in the x direction and the number m of subdivisions in the y direction. Here is an example.

```
    f = inline(' exp( x + cos(y.^2))', 'x', 'y')
    corners = [1,2, 3,4]
    I1 = simp2(f,corners)
    enter the number of subdivisions . . .
Approximate value of the integral using Simpson's rule
I1 = 5 .4490
```

Make a short Mfile for each problem that contains all the commands that you used. Use the command `diary` and the command `type` to print out the Mfile. See the end of Chapter 2 of the *MATLAB Companion*. It is on my webpage. For each problem, hand in the diary file.

1. Let R be the rectangle $\{0 \le x \le 2,\ 1 \le y \le 4\}$.

a) Let $f(x,y) = x\cos(x^2+y)$. Calculate the integral $\int\int_R f\,dA$ by hand to get the exact value. Call this value I_0.

b) Use the Mfile `simp2` to estimate the integral with $[n,m] = [40,60]$. Call the result I_1. Then estimate it again with $[n,m] = [80,120]$ and call the result I_2. How well does the difference $|I_1 - I_2|$ approximate the actual error $I_1 - I_0$?

2. Find the volume of the region that is the intersection of the solid cylinder $\{(x-1)^2 + y^2 \le 1/4\}$ with the ball $\{x^2+y^2+z^2 \le 4\}$. First make a change of variable $\tilde{x} = x - 1$ to move the cylinder so that its axis of symmetry is the z axis. Then put the integral into polar coordinates. Finally, use `simp2` to estimate the integral with an error on the order of 10^{-6}.

3. Find the area of the surface $z = \exp(-x^2 - y^2)$ over the triangle $G = \{0 \le x \le 1,\ 0 \le y \le x\}$.

a) Express the surface area as a double integral over G,

$$\text{surface area } = \int\int_G f(x,y)\ dA(x,y)$$

for an appropriate function $f(x,y)$.

b) Make a change of variable $(u,v) \to (x(u,v), y(u,v))$ that maps the rectangle $R = \{0 \le u \le 1,\ 0 \le v \le 1\}$ onto G. Then using the change of variable for integrals, the integral in part a) becomes

$$\int\int_R f(x(u,v), y(u,v))|J(u,v)|\ dudv.$$

c) Estimate the double integral over R with an error on the order of 10^{-4} or less using `simp2`.

4. Let C be the helix parameterized by $\mathbf{r}(t) = (\cos t,\ \sin t,\ \log(1+t))$, $0 \le t \le 2\pi$. Suppose that C is a wire composed of a material with linear density $\rho(x,y,z) = \exp(x^2 - y^2 - z)$.

a) Write an integral expression for the mass of the wire C.

b) Make a numerical estimate of the integral using the MATLAB integrator `quadl`. To see how to use this code, enter `help quadl`.