MATH 241
MATLAB Problem Set 4

This problem set is based on material in chapter 11 in the *MATLAB Companion*. You will need to download the three MATLAB Mfiles `lint.m, curl.m` and `flux2.m` from the web. These Mfiles are contained in the Mfiles for the MATLAB Companion. The web address is `www.math.umd.edu/~jec`. To see how to use `lint.m`, enter `type lint` at the MATLAB prompt. The directions for use are at the beginning of the file. The same applies for `curl.m` and `flux2.m`

1. Let C be the helix parameterized by $\mathbf{r}(t) = (\cos t,\ \sin t,\ \log(1+t))$, $0 \le t \le 2\pi$. Suppose that C is a wire composed of a material with linear density $\rho(x, y, z) = \exp(x^2 - y^2 - z)$.

a) Write an integral expression for the mass of the wire C.

b) Make a numerical estimate of the integral using the MATLAB integrator `quadl`. To see how to use this code, enter `help quadl`.

2. Let $\mathbf{F}(x, y) = (y^2, x/10)$ be a two-dimesional vector field. Use the Mfile `lint.m` to do the following calculations on the rectangle $R = \{0 \le x \le 1, 1 \le y \le 2\}$.

a) Use just one segment from $P_1 = (.2, 1.2)$ to $Q_1 = (.8, 1.8)$. What is the value of the line integral $\int_P^Q \mathbf{F} \cdot d\mathbf{r}$?

b) Use just one segment from $P_2 = (.8, 1.2)$ to $Q_2 = (.2, 1.8)$. What is the value of the line integral this time? How do you explain the difference in signs between the result of part a) and that of part b)? Look at the angle between the vector $\mathbf{F}$ and the tangent vector the segment.

c) Now use four segments, tracing out roughly a square with the vertices taken in this order P_1, P_2, Q_1, Q_2 and back to P_1. On which segment is the integral positive? On which is it negative? On which segments is it very small? Explain.

3. Let the vector field be $\mathbf{F} = (\sin(xy), x - y)$ and let the square $R = \{0 \le x \le 2.5, 0 \le y \le 2.5\}$. Set the vector `corners = [0 2.5 0 2.5]`.

a) Let C be the path consisting of the two straight segments from $P = (.5, .5)$ to $(1.5, 1)$, and from $(1.5, 1)$ to $Q = (2, 2)$. Use `lint` to compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

b) Repeat the calculation with a different path $\tilde{C}$ from P to Q. Use any number of segments. Are the results from part a) and b) the same? Is this vector field conservative?

c) Calculate the line integral in the counterclockwise direction around a small triangle, that is centered at the point $(2, 2)$. Then from the information displayed on the screen, calculate the ratio (total line integral)/area. Compare the ratio you calculated with the value of the curl (calculated by hand) at the point $(2,2)$. To make the values closer, use a smaller triangle.

If you want to see the values of the curl displayed in a color map, use the command `curl(u,v,corners)`.

4. Let $\mathbf{F}(x,y) = (x + \sin y, x\cos y)$. Let the rectangle $R = \{1 \le x \le 4, -2 \le y \le 2\}$.

a) Use the Mfile `lint.m` to calculate the line integral along any two paths from the point $P = (1.5, -1)$ to the point $Q = (3.5, 1)$. Use any number of segments. What do you observe?

b) Calculate the line integral around any closed path. What can you conclude about this vector field? Verify your conclusion with a hand calculation.

5. Let $\mathbf{F}(x,y) = (1, y\sin x)$ and let the square $R = \{-2 \le x \le 2, -2 \le y \le 2\}$

Note: To make an inline function for $u(x,y) \equiv 1$, you must write it `u = inline('1+0*x','x', 'y')`.

a) Calculate $\text{div}(\mathbf{F})$ by hand. Where is it positive and where is it negative?

b) Use the Mfile `flux2.m`. It works in the same way as `lint.m`. Calculate the flux integral in the counterclockwise direction around a small triangle that is centered at the point $(-1,0)$. Divide the total flux by the area of the triangle. Compare it with the value of $\text{div}(\mathbf{F})$ at the point$(-1,0)$.

c) Do the same calculation as in part b) for a triangle centered at the point $(1,1)$. From the appearance of the vector field, what do you expect the sign of $\text{div}(\mathbf{F})$ to be at $(1,1)$? Explain.