

1. Let $f(x, y) = y^2/x + yx^2$.

a) If we move in the direction $\mathbf{u} = (\mathbf{i} + \mathbf{j})/\sqrt{2}$ from the point $(2, 1)$, does the function f increase or decrease?

b) In what direction should we move from $(2, 1)$ to get the maximum rate of increase? What is the maximum rate of increase?

c) What is the direction of a tangent vector to the level curve of f through the point $(2, 1)$?

2. a) Let $f(x, y, z)$ be given with

$$f_x(1, 0, 2) = 1, \quad f_y(1, 0, 2) = -2, \quad f_z(1, 0, 2) = 3.$$

Let $x(t) = t^2$, $y(t) = \sin(\pi t)$, $z(t) = 2t$. Set $F(t) = f(x(t), y(t), z(t))$. What is $dF(t)/dt$ at $t = 1$?

3. Find the maximum and minimum values of $f(x, y) = (x - 1)^2 + 2y^2$ on the set

$$R = \{(x, y) : x^2 + y^2 \leq 1\}.$$

Sketch the set R and the level curves of f .

4. Write the following iterated integral as a double integral over a set R in the x, y plane. Sketch the set R . Then evaluate the integral by changing the order of integration.

$$\int_0^1 \int_{y^{1/3}}^1 y dx dy.$$

5. Let D be the solid region which lies inside the sphere $x^2 + y^2 + z^2 = 4$, and outside the cylinder $x^2 + y^2 = 1$.

a) Sketch the intersection of D with the xy plane.

b) Find the volume of D . Use the symmetry of the region to write the integral in polar coordinates. Calculate the integral.