MATH 401 Applications of Linear Algebra

Matlab Problem Set 1

Problems

1. Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 5 & -2 & 0 \\ 2 & 4 & -2 & 1 \\ 4 & 2 & -5 & 1 \end{bmatrix}.$$

a) Use the MATLAB command `[L,U,P] = lu(A)` to get the factorization $PA = LU$. Verify that indeed `P*A = L*U`.

b) Now let $B = PA$. Try the command `[L,U,P] = lu(B)`. Why is $P = I$?

2. In this exercise we see the speed gained by using the LU factorization. For a given vector x_1 we want to find the vectors x_k defined by the recursive relation

$$Ax_{k+1} = x_k, \qquad k = 1, \ldots, 49.$$

We shall do this in a Mfile as follows:

```
 % create a random 100 x 100 matrices with
   entries between 0 and 1
 A = rand(100,100);
 % create a column vector of all ones.
 x1 = ones(100,1);
 x = x1;

tic
for k = 1:49
   x = A\x;
end
toc
% displays the time elapsed to compute
  the 49 iterates.
% this is x_50
x
```

Now extend this Mfile by writing the command that factors A, $PA = LU$, and then write a loop that solves the equations

$$Ly = Px_k \quad Ux_{k+1} = y, \quad k = 1, \ldots, 49.$$

Put a command `tic` before the factorization command, and put a `toc` command at the end of the loop. Compare the last values of x in both loops to make sure you are doing the same calculation. Compare the times needed for the different loops for several different matrices A. Each time you run the Mfile, a different matrix A is created.

3. In this exercise we use MATLAB to solve the equations used to describe the system of springs and masses. We will work with a system of 7 springs and 6 masses. We shall assume that the springs all have length $l = .1$ Let the spring constants be $k_j, j = 1, \ldots, 7$. Let $a = (a_1, a_2, \ldots, a_6)$ be the equilibrium positions of the joints of the springs when then there are no masses. They satisfy the equations $Sa = b$ where S is the tridiagonal matrix constructed from the spring constants

$$S = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & 0 & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 & -k_5 & 0 \\ 0 & 0 & 0 & -k_5 & k_5 + k_6 & -k_6 \\ 0 & 0 & 0 & 0 & -k_6 & k_6 + k_7 \end{bmatrix}$$

and b is the column vector

$$b = (l(k_1 - k_2),\ l(k_2 - k_3),\ l(k_3 - l_4),\ l(k_4 - k_5),\ l(k_5 - k_6),\ l(k_6 - k_7) + k_7).$$

a) Write a script Mfile that does the following:

(i) Takes a vector of values k_j as input;
(ii) Creates the matrix S and the vector b;
(iii) Solves the system $Sa = b$; and
(iv) Plots the joining points a_j on the x axis.

Try various vectors of spring constants: First $k = (1, 1, 1, 1, 1, 1, 1)$. Then $k = (1, 2, 3, 4, 3, 2, 1)$. Finally take $k = (1, 2, 3, 4, 5, 6, 7)$. Give a physical interpretation of your results in each case. Verify in each case that $a_1 \geq l$, $a_j - a_{j-1} \geq l$, and $1 - a_6 \geq l$. How do you interpret this physically?

Now consider the system that describes the small displacements of the masses. Let

$$d_j = a_j - a_{j-1}, \quad j = 2, \dots, 6$$

while

$$d_1 = a_1, \qquad d_7 = 1 - a_6.$$

The numbers d_j are the lengths of the stretched springs in equilibrium. We set

$$r_j = 1 - \frac{l}{d_j}$$

Since all $d_j > l$, the numbers $r_j > 0$. Let T be the matrix obtained by replacing k_j in S by $r_j k_j$.

Let $m = (m_1, \dots, m_6)$ be the vector of masses. Then the vector $y = (y_1, \dots, y_6)$ of small displacments of masses satisfies

$$Ty = -gm$$

where g is the gravitational constant, $g = 9.8$ m/s.

b) Extend the Mfile of part a) to do the following:

(i) Accepts a vector $m = (m_1, \dots, m_6)$ of masses as input;
(ii) Creates the matrix T and the vector $-gm$;
(iii) Solves the system $Ty = -mg$;
(iv) plots the solution points (a_j, y_j)

Set all the $k_j = 1$, and try various combinations of masses. First try all $m_j = .001$. Next try $m_1 = m_2 = m_3 = m_6 = .01$, and $m_4 = m_5 = .003$.