Final Examination – Math 140 – Fall 2008

Answer each of the 10 numbered problems on a separate answer sheet. Each answer sheet must have your name, your TA's name, and the problem number (= page number). Show all your work for each problem clearly on the answer sheet for that problem. You must show enough written work to justify your answers. No calculators or electronic equipment or cell phones are allowed. Don't forget to sign the Honor Code on the last of the answer sheets. Good luck!

1. (10 points each) For each of the limits in parts (a) - (c), determine whether the limit exists as a number, as ∞ or $-\infty$, or does not exist. If the limit is a number, evaluate it. Give reasons.

   (a) $\lim_{x\to 0} \dfrac{\sqrt{x+7}-\sqrt{7}}{x}$ (b) $\lim_{t\to 0+} \dfrac{\sin(4t)}{t\cos(t)}$ (c) $\lim_{z\to -\infty} \dfrac{7e^{-z}+e^{z}}{4e^{-z}+2e^{z}}$

2. (10 points each) Compute the following derivatives. (Don't simplify your answers.)

   (a) $\dfrac{d}{dt}\dfrac{te^{2t}}{3t^4+5}$ (b) $\dfrac{d}{dz}(\cos^3(\pi z)+4)^{-5}$ (c) $\dfrac{d}{dx}\displaystyle\int_{\sin x}^{2\pi} \ln(1+t^2)\,dt$

3. (a) (10) Consider the equation $2x^4 - 6 = xy^3 + y^2$. Show that the point $(1,-2)$ is on the graph of the equation, and find an equation of the line L that is tangent to the graph of the equation at the point $(1,-2)$.

   (b) (10) An object moves along the horizontal axis with position $h(t) = t^2 \ln(3t)$ for $t \geq 4$, where t is registered in minutes. Find the acceleration $a(6)$ of the object when $t = 6$ (minutes).

4. Let $f(x) = x^5 + 4x^3 + 2x + 3$, for all x.

   (a) (5) Explain why there is a solution z^* to the equation $f(z^*) = 0$.

   (b) (5) Choose a reasonable value of c_1 for the Newton-Raphson method, and use the associated formula to obtain the next approximation c_2 to the solution z^*.

5. Let $f(x) = \dfrac{x-3}{(x-2)^3}$, so that $f'(x) = \dfrac{7-2x}{(x-2)^4}$ and $f''(x) = \dfrac{6(x-4)}{(x-2)^5}$.

   (a) (5) Find the domain and intercepts of f, identifying each.

   (b) (5) Find the asymptotes of f, identifying each.

   (c) (5) Find the intervals on which f is increasing (if any), and locate all extreme value(s) of f.

   (d) (5) Find all intervals on which the graph of f is concave upward (if any).

   (e) (5) Find all inflection points (if any).

   (f) (10) Sketch a graph of f, indicating all pertinent information on the graph.

6. (10) A board 10 feet long slides down a wall. At the instant the bottom end is 8 feet from the wall, the other end is moving along the wall at the rate of -5 feet per second. Determine how fast is the bottom end sliding across the ground at that moment. Include a picture of the situation.

7. A rectangle with sides parallel to the x and y axes is to be inscribed in the ellipse $x^2 + 4y^2 = 4$.

   (a) (5) Sketch a graph of the ellipse and a representative rectangle.

   (b) (10) Find the area A of the rectangle with maximum possible area. Give reasons why you have found the maximum area.

8. (10 points per part) Perform the integrations:

   (a) $\displaystyle\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$ (b) $\displaystyle\int \frac{\sec^2(3t)}{\tan(3t)}\,dt$

9. (a) (10) Evaluate $\displaystyle\int_0^3 |4-2z|\,dz$.

   (b) (10) A parabola has directrix $y = -4$ and focus $(0, 0)$. Find an equation for the parabola, and sketch its graph.

10. (10) Let $f(x) = x^2 - 4x - 5$ and $g(x) = x^3 - 3x^2 - 5$. Find the area A of the bounded region determined by the graphs of f and g.

NOTE: Please don't forget to sign the Honor Pledge on page 10.