Math 140 – Final Examination

May 15, 2008

Answer each of the 7 numbered problems on a separate answer sheet. Each answer sheet must have your name, your TA's name and the problem number (= page number). Show all your work for each problem clearly on the answer sheet for that problem. You must show enough work to justify your answers. When you have finished the exam, arrange the answer sheets in order and write and sign the Honor Code on the first answer sheet (only the first one). **No calculators or electronic equipment allowed. Please turn off your cell phone.**

1. (10 each) For each of the limits in parts (a)-(c) determine whether the limit exists as a number, as ∞ or $-\infty$, or does not exist. If the limit is a number, evaluate it. Give pertinent reasons.

   (a) $\displaystyle\lim_{x\to-3} \frac{\sin(2x+6)}{x+3}$

   (b) $\displaystyle\lim_{h\to0} \frac{|2h-10|}{5-h}$

   (c) $\displaystyle\lim_{t\to-5} \ln(t+5)^2$

2. (a) (12) Given the equation $x^5 + x = 3$, find an interval in the form $[n, n+1]$, where n is an integer, that must contain the solution. Support your choice of interval by showing your work and quoting an appropriate theorem.

   (b) (18) Consider the equation $9x^2 + y^2 = 36$.

      i. Sketch and name the graph of the equation. Remember to label intercepts with their values or coordinates and to label the axes.

      ii. Using this equation, which defines y implicitly as a function of x, find $\dfrac{dy}{dx}$ (without solving for y).

      iii. Find an equation for the line tangent to the graph of the equation at the point $(-1, -3\sqrt{3})$.

3. (a) (10) A cone's base radius and height are constantly changing. Suppose when the radius of the base is 3 cm and the height 6 cm, the height is increasing at a rate of 1 cm/s and the volume is decreasing at a rate of π cm^3/s. How fast is the base radius changing at that time?

   (b) (16) A particle is moving along a straight line. Its position at time t is given by the function $s(t) = \cos^2(\pi t)$. Find the velocity, $v(t)$, and acceleration, $a(t)$, at time t. You do not have to simplify the expression for $a(t)$.

4. Given $f(x) = \dfrac{x-2}{\sqrt{x^2+2}}$. Then $f'(x) = \dfrac{2x+2}{(x^2+2)^{3/2}}$, and $f''(x) = \dfrac{-2(2x^2+3x-2)}{(x^2+2)^{5/2}}$. Find each of the following. If an item does not exist, answer NONE.

   (a) (10) $\displaystyle\lim_{x\to\infty} f(x)$ and $\displaystyle\lim_{x\to-\infty} f(x)$ (Caution: These two limits have different values.)

   (b) (6) Vertical asymptote(s) and horizontal asymptote(s)

   (c) (6) Interval(s) where f is increasing and the interval(s) where f is decreasing.

   (d) (3) Maximum value and where it occurs.

   (e) (3) Minimum value and where it occurs.

   (f) (6) Interval(s) for which the graph of f is concave upward and on interval(s) for which it is concave downward

Please turn over for Problems 5, 6 and 7.

5. (a) (10) Sketch the graph of a function f defined on $(-\infty, 1)$ and on $(1, \infty)$, with the following properties. Label the graph with all pertinent information.

   i. $\lim_{x\to-\infty} f(x) = -1$ and $\lim_{x\to\infty} f(x) = 2$.

   ii. $\lim_{x\to 1^-} f(x) = \infty$ and $\lim_{x\to 1+} f(x) = \infty$.

   iii. The graph of f is concave upward on $(-\infty, 1)$ and on $(1, 5)$, and is concave downward on $(5, \infty)$.

   iv. $f(3) = 0$ is a relative minimum value of f.

   (b) (15) Set up the iterated integral(s) needed to find the area between the graphs of $f(x) = x^3+x^2+2x+1$ and $g(x) = x^2 + 6x + 1$. The integrand should be simplified and the integrals should be ready for integration but DO NOT EVALUATE.

6. (10 each) Evaluate each of the following.

   (a) $\int_{-2}^{4} |x+1|\, dx$

   (b) $\int [\sin(3x) + e^{-x} - 3\cos(x)]\, dx$

   (c) $\int \frac{t^2}{\sqrt[3]{1-t}}\, dt$

7. (a) (10) Let $G(x) = \int_{e^x}^{e^{3x}} (\ln(t)+6)^{1/2}\, dt$. Find $G'(x)$. Rewrite your answer so that there are no ln's in it.

   (b) (15) A rectangular box with a square base and no top is to be made using a total of 12 square feet of cardboard. Find the dimensions of the box of largest possible volume V. Include a picture of the situation.

End of exam.