

SHOW ALL YOUR WORK. Write your name and your TA's name on each answer sheet. Number the answer sheets from 1 to 7. Work problem 1 on page 1, etc. Use the back of the same answer sheet if necessary for extra space. Each problem is worth the indicated 30 points except for number 5 which is worth 20. No calculators or other electronic devices of any type. Turn off your cell phone and put it away.

1. Use Theorems on Limits, referring to which you use, to determine

$$(i) \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}, \quad (ii) \lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(5x)}.$$

2. Find the derivative of each of the three functions

$$f(x) = 6x + 3x^{7/3}, \quad g(x) = (1 - \cos(x))^{1/2}, \quad h(x) = 4^{2x-1}.$$

and state where each function is differentiable.

3. (i) Find y' in terms of x, y for the implicit equation

$$x^3 + 3x^2y^2 + y^3 = 5.$$

(ii) The graph passes through $(1, 1)$. Find the equation of the tangent line at this point.

4. Water leaks onto a floor forming a pool in the shape of a cylinder of height 1 mm with volume increasing at a rate of $4 \text{ cm}^3/\text{min}$. How fast is the radius of the pool increasing when the radius is 12 cm.
5. Show that the function $f(x) = x^5 - 4x^4 + 2x^3 + 2x^2 + 3x - 5$ has an inflection point in the interval $[0, 1]$.

6. Evaluate

$$(i) \lim_{h \rightarrow 0} \frac{1}{h} \int_0^h 5e^{-x^2} dx \quad (ii) \lim_{t \rightarrow 0^+} \frac{\sqrt{1+t^3}}{\sqrt{t}}.$$

7. Compute

$$\int 2x \sqrt{x+3} \, dx, \quad \int_3^8 |x-6| \, dx$$