Math 140, Jeffrey Adams
Test I, September 18, 1998

IMPORTANT INSTRUCTIONS
1. Write your name, section number, and TA's name on each answer sheet.
2. Number the sheets 1-5. Do all of the work for problem 1 on sheet 1. You may use the back if necessary – write "*see back of sheet*". Similarly for problems 2-5.
3. For full credit you must **show your work**.

Question 1. (20 points) Let $f(x) = x^3 + 1$.
(a) By taking a limit, find the slope of the tangent line to the graph at the point $(1, 2)$.

(b) Find an equation of the tangent line to the graph at the point $(1, 2)$.

Question 2. (20 points) Evaluate each of the following limits. Your answer should be either a number, $+\infty$ or $-\infty$, or "does not exist". *Show your work.*

(a) $\lim_{x\to 0} \dfrac{\sin(2x)}{3x}$

(b) $\lim_{x\to 1} \dfrac{2x-2}{x^2-1}$.

(c) $\lim_{x\to\infty} \dfrac{x^3-2}{x^2+2x-1}$

(d) $\lim_{x\to 1^+} \dfrac{|x-1|}{1-x}$.

Question 3. (20 points) Let $f(x) = \begin{cases} x & 0 \le x \le 2 \\ x^2 - 1 & 2 < x \end{cases}$.

(a) Is f continuous from the left at 2? Continuous from the right at 2? Continuous at 2? Justify your answers.

(b) Is f continuous on the closed interval $[0, 1]$?. Justify your answer.

Question 4. (20 points) Use the intermediate value theorem to prove that the equation $\cos(x) = x$ has a solution $x \ge 0$. (It is not sufficient to give a graphical or calculator solution.)

Question 5. (20 points) Suppose the position of a moving particle is given as a function of time by $f(t) = t^2 + 3t - 1$.

(a) Write down the formula involving a limit which gives the velocity of the particle at time $t = 1$.

(b) Evaluate the limit to find the velocity at $t = 1$.