Math 140, Jeffrey Adams/Test II SOLUTIONS

Question 1. (20 points) (a)

$$f'(x) = \frac{(\cos(2x) + x(-2\sin(2x)))e^{3x} - x\cos(2x)3e^{3x}}{e^{6x}}$$

Plugging in $x = 0$ gives $f''(0) = \frac{(1+0)1-0}{1} = 1$.

(b)

$$f'(x) = \frac{1}{\cos(x)}(-\sin(x)) = -\frac{\sin(x)}{\cos(x)} = -\tan(x),$$

and

$$f''(x) = -\frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = -\frac{1}{\cos^2(x)} = -\sec^2(x).$$

Question 2. (20 points)

(a) Differentiating gives $\sin(x-y)(1-\frac{dy}{dx}) = 1$. Solve this for $\frac{dy}{dx}$: $1-\frac{dy}{dx} = -\frac{1}{\sin(x-y)}$, or $\frac{dy}{dx} = 1 + \frac{1}{\sin(x-y)}$.

(b) Plugging in $x = 0, y = \pi/2$ gives $\frac{dy}{dx} = 1 + \frac{1}{\sin(-\pi/2)} = 1 - 1 = 0$.

(c) Since the the tangent line has slope 0 by (b), it is horizontal, with equation $y = \pi/2$. Alternatively, $y - \frac{\pi}{2} = 0(x - 0)$, or $y = -\frac{\pi}{/}2$.

Question 3. (20 points)

Let O be the center of the circle, P be the point on the circle. Let L be a horizontal line through the center of the circle. Let θ be the angle between the line OP and L, and let h be the height of P above the ground.

Then $\frac{d\theta}{dt} = 3$, and we are looking for $\frac{dh}{dt}$. The equation relating them is $h = 100\sin(\theta)$. Therefore $\frac{dh}{dt} = 100\cos(\theta)\frac{d\theta}{dt} = 300\cos(\theta)$.

We need to find $\cos(\theta)$ when $h = 50$. When $h = 50$, $\sin(\theta) = 50/100 = \frac{1}{2}$. Therefore $\cos(\theta) = \sqrt{1-\sin^2(\theta)} = \sqrt{1-\frac{1}{4}} = \sqrt{3}/2$. Therefore $\frac{dh}{dt} = 300\sqrt{3}/2 = 150\sqrt{3} = 259.8\ldots$.

Question 4. (20 points)

(a) Let $f(x) = \ln(x)$, $a = 20$ and $h = .01$. Then $f(20.01) \simeq f(20) + (.01)f'(20)$. Since $f'(x) = \frac{1}{x}$, $f'(20) = \frac{1}{20} = .05$. Therefore

$$f(20.01) \simeq 2.9957322735 + (.01)(.05) = 2.9962322735.$$

(b) The error is $2.9962322735 - 2.9962321485 = .00000001249$ (or $.0000001250$, depending on how your round off.)

Question 5. (20 points)

The first step is $c_1 = 4 - \frac{f(4)}{f'(4)} = 4 - \frac{Log(4)-4+3}{\frac{1}{4}-1} = 4.5150591481$ and the second is

$$\begin{aligned} c_2 &= 4.5150591481 - \frac{f(4.5150591481)}{f'(4.5150591481)} \\ &= 4.5150591481 - \frac{Log(4.5150591481) - 4.5150591481 + 3}{\frac{1}{4.5150591481} - 1} = 4.5052445368 \end{aligned}$$