Math 140, Jeffrey Adams
Test II, March 10, 2010 SOLUTIONS

Question 1.
(a)

$$\begin{aligned} f'(x) &= \lim_{t\to x} \frac{(t^2+2)-(x^2+2)}{t-x} \\ &= \lim_{t\to x} \frac{(t^2-x^2)}{t-x} \\ &= \lim_{t\to x} \frac{(t-x)(t+x)}{t-x} \\ &= \lim_{t\to x}(t+x) = 2x \end{aligned} \tag{1}$$

(b) When x is near 1 $f(x) = 2x+1$, so

$$f'(x) = \lim_{x\to 1} \frac{(2x+1)-3}{x-1} = \lim_{x\to 1} \frac{2x-2}{x-1} = \lim_{x\to 1} \frac{2(x-1)}{x-1} = 2 \tag{2}$$

(c) Let $g(x) = |x|$ and $h(x) = x^2 - 1$. Then $f(x) = g(h(x))$ so by the chain rule $f'(x) = g'(h(x))h'(x)$ provided $g'(h(x))$ and $h'(x)$ exists. Obviously $h'(x)$ exists everywhere, so the only question is $g'(h(x))$. Since $g(x) = |x|$ is differentiable except at $x = 0$, $g'(h(x))$ exists unless $h(x) = 1$, i.e. $x^2 - 1 = 0$, i.e. $x = \pm 1$. So $f(x)$ is differentiable for all $x \neq \pm 1$.

This is also easy to see from the picture: the graph has corners at ± 1.

Question 2.

(a) $\frac{d}{dx}\sin(x^2+x) = \cos(x^2+x)(2x+1)$

(b) $\frac{d}{dx}\ln(\cos(x^2)) = \frac{1}{\cos(x^2)}(-\sin(x^2)(2x))$

(c) $\frac{d}{dx}(\frac{x^2+1}{x^3+1}) = \frac{(2x)(x^3+1)-(x^2+1)(3x^2)}{(x^3+1)^2}$.

(d) $\frac{d^2}{dx^2}(xe^x)$: $\frac{d}{dx}(xe^x) = e^x + xe^x$, and $\frac{d^2}{dx^2}(xe^x) = \frac{d}{dx}(e^x + xe^x) = e^x + e^x + xe^x = 2e^x + xe^x$.

Question 3.
(a) By implicit differentiation:

$$\cos(y) + x(-\sin(y))\frac{dy}{dx} + \frac{dy}{dx} = 0 \tag{3}$$

so

$$\cos(y) + \frac{dy}{dx}(-x\sin(y)+1) = 0 \tag{4}$$

or

$$\frac{dy}{dx} = \frac{-\cos(y)}{-x\sin(y)+1} \tag{5}$$

and plugging in $x = 0, y = \pi$ gives $\frac{dy}{dx} = -\cos(\pi) = 1$.

(b) Label the distance from the wall to the foot of the latter $x(t)$, and the position of the top by $y(t)$. Then $\frac{dx}{dt} = 8$, and we want $\frac{dy}{dt}$. Taking $\frac{d}{dt}$ of $x^2 + y^2 = 5$ gives $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$.

$$\frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{2y}. \tag{6}$$

Take $\frac{dx}{dt} = 8$. Also when $y = 4$ then $x = 3$ since $x^2 + y^2 = 5^2$, so

$$\frac{dy}{dt} = -\frac{3 * 8}{4} = -6.$$

Question 4.
(a)

$$\begin{aligned} c_2 &= c_1 - \frac{f(c_1)}{f'(c_1)} \\ &= 1 - \frac{3}{3+2} \\ &= \frac{2}{5} \end{aligned} \tag{7}$$

and

$$\begin{aligned} c_3 &= c_2 - \frac{f(c_2)}{f'(c_2)} \\ &= \frac{2}{5} - \frac{(\frac{2}{5})^2 + (\frac{2}{5}) + 1}{3(\frac{2}{5})^2 + 2(\frac{2}{5})} \end{aligned} \tag{8}$$

(b) Let $f(x) = \sqrt[4]{x} = x^{\frac{1}{4}}$, so $f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$. Take $a = 16$. Then $f(a) = 2$ and $f'(a) = \frac{1}{4}16^{-\frac{3}{4}} = \frac{1}{4*16^{\frac{3}{4}}} = \frac{1}{4*(16^{\frac{1}{4}})^3} = \frac{1}{4*2^3} = \frac{1}{32}$. So, with $x = 17$,

$$\sqrt[4]{17} \sim f(a) + f'(a)(x-a) = 2 + \frac{1}{32}(1) = 2\frac{1}{32}. \tag{9}$$