Math 140, Jeffrey Adams

Test III, November 12, 1999

IMPORTANT INSTRUCTIONS

1. Write your name, section number, and TA's name on each answer sheet.

2. Number the sheets 1-4. Do all of the work for problem 1 on sheet 1. You may use the back if necessary – write "see back of sheet". Similarly for problems 2-4. **ANSWER ONLY ONE PROBLEM ON EACH PAGE**

3. For full credit you must **show your work**.

4. No Calculators

Question 1. (25 points)

(a) Suppose $g(x) = x^2 - \sin(x)$. Find all functions f(x) for which f'(x) = g(x), and the one for which f(0) = 3.

(b) Find the maximum value of the function $f(x) = x\sqrt{6-x}$.

Question 2. (25 points)

(a) Evaluate $\lim_{x\to\infty} \frac{5+12\sqrt{x}}{7-3\sqrt{x}}$. Justify your answer.

(b) Find the value of the constant c so that $\lim_{x\to\infty} \frac{cx^3+1}{(x+1)(2x+1)(3x+1)} = 1.$

Question 3. (25 points)

Let
$$f(x) = \frac{16x^2}{8+x^3}$$
, so that $f'(x) = \frac{16x(16-x^3)}{(8+x^3)^2}$ and $f''(x) = \frac{32(64-56x^3+x^6)}{(8+x^3)^3}$

(a) Determine the horizontal and vertical asymptotes of the graph of f(x), if any. Determine the x and y intercepts of the graph of f(x), the region where f(x) positive, and the region where f(x) is negative.

(b) Draw a rough sketch of the graph of f(x) based on the information so far.

(c) Find any critical points of f(x), and determine whether f(x) has a relative (i.e. local) maximum, relative minimum, or neither at the critical points.

(d) Given the additional information that $f''(1) = \frac{32}{81}$, $f''(2) = -\frac{5}{2}$ and $f''(3) = -\frac{23008}{42875}$, and $f''(4) = \frac{4}{81}$, give a more accurate sketch of the graph over the interval [1, 4]. Indicate where the relative maximum occurs and approximately where (between 1 and 2, between 2 and 3, or between 3 and 4) the inflection points occur.

Question 4. (25 points) Suppose an amount $f(t) = ce^{kt}$ of radioactive material decreases to $\frac{1}{4}$ the original amount in 15 days. (You may leave your answers in terms of ln.)

(a) Find k.

(b) Find the time it takes for the amount to decrease to 5% of the original amount.

(c) What amount of the material would you need to start with so that after 20 days there is 12 grams of the material remaining?

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