Math 141 **Final Examination** Spring, 2003

Instructions: Answer each of the 11 numbered problems on a separate answer sheet. Each answer sheet must have your name, your TA's name, and the problem number (=page number). Show all your work for each problem clearly on the answer sheet for that problem. You must show enough written work to **justify your answers.**

NO CALCULATORS

1. (10 points EACH)
   a) Compute $f'(e)$ when $f(x) = x^{-x}$.
   b) Compute $\int_1^2 (\ln x)^2\, dx$.
   c) Evaluate $\int \dfrac{1}{x^2 + 2x - 3}\, dx$.

2. (10 points EACH) Let $f(x) = 3x \ln x$.
   a) Find the smallest number a so that the restriction of f to (a, ∞) has an inverse. Let h denote the inverse of f restricted to this interval.
   b) Compute $h'(0)$, where h is defined in part (a).

3. (10 points EACH)
   a) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^{3n}}{3^{2n+1}}$
   b) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$ converge or diverge? EXPLAIN your answer.

4. (15 points) Find the radius of convergence of the series $\displaystyle\sum_{n=2}^{\infty} \frac{2^n x^n}{(\ln n)^2}$.

5. (15 points) Suppose that y is a function of x, satisfies the differential equation $y' = \dfrac{xy}{x+2}$, and $y(-1) = 1$. Compute $y(0)$.

Exam continues on reverse ——>

6. (10 points EACH) Recall that $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for certain values of x.

   a) Find a power series for the function $f(x) = \dfrac{x}{1-4x}$

   b) Use the power series in (a) to compute $f^{(6)}(0)$, the sixth derivative of f evaluated at 0.

7. (15 points) Find the length L of the parametrized curve given by the equations

$$x = e^t \cos t, \quad y = e^t \sin t$$

   for $0 \leq t \leq 1$.

8. (15 points) Let R be the region bounded below by the x-axis, on the left by the y-axis, and above by the graph of $y = \sqrt{1-x}$. Find the vertical coordinate $\bar{y}$ of the center of gravity of R.

9. (15 points) Let R be the region inside the rectangle with vertices at $(3,1)$, $(6,1)$, $(3,3)$, and $(6,3)$. Find the volume V of the solid region formed by revolving R around the x-axis.

10. (15 points) Determine whether the improper integral $\displaystyle\int_0^3 \frac{2x}{(1-x^2)^2}\,dx$ converges or diverges, giving reasons. If it converges, then compute its value.

11. (10 points EACH)

    a) Find the area A of one of the 4 'petals' of the polar graph $r = \cos 2\theta$.

    b) Evaluate $\displaystyle\int \frac{1}{\sqrt{16-25x^2}}\,dx$.

END OF EXAM – GOOD LUCK!