MIDTERM 4 — MATH 141 — SPRING 2002 — BOYLE

No books, no notes, no calculators. Use exactly one page for each of the four numbered questions (use the back of the page if necessary).
Put your name, your TA's name and the question number on each page.
Put a box around the final answer to a question.

1. (20 points) Suppose a water tank has the shape formed by rotating the parabola $y = x^2$, $0 \leq x \leq 1$, about the y-axis. Assume x and y are given in feet and water has a density of 62.5 pounds per cubic foot. Let W denote the number of foot-pounds required to pump the water over the top of the tank.
   Write an integral which computes W. Do not evaluate the integral.

2. (a) (10 points) Suppose the position in the plane of a particle at time t is given by the equations $x(t) = \cos(5t)$, $y(t) = 3\sin(5t)$.
   What is the particle's speed when $t = \pi/10$?

(b) (15 points) Compute the arc length of the polar coordinates graph $r = \theta^2$, $0 \leq \theta \leq \pi/2$.

3. (25 points) (Polar coordinates) Let S denote the region bounded by the graph of $r = 1$. Let T be the region bounded by the graph of $r = 2\sin(\theta)$. Compute the area of the region in the first quadrant which lies inside S and outside T.

4. (a) (20 points) Let R denote the region between the x-axis and the graph of

$$f(x) = \sqrt{\arctan(x)} , \quad 0 \leq x \leq 1 \ .$$

Compute the volume of the solid obtained by revolving R around the x-axis.

(b) (10 points) Use the relationship between $\cos(\theta)$, $\sin(\theta)$ and $e^{i\theta}$ to prove that $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, whenever a and b are real numbers.