**Second In-Class Exam**
**Math 246, Professor David Levermore**
**Tuesday, 19 October 2017**

**Your Name:** ________________________________

**UMD SID:** ________________________________

**Discussion Instructor (circle one):** Yan Tay Jing Zhou
**Discussion Time (circle one):** 8:00 9:00 10:00

**No books, notes, calculators, or any electronic devices.** If you need more space to answer a problem then use the back of one of these pages. Clearly indicate where your answer to each part of every problem is located. **Your reasoning must be given for full credit.** Any work that you do not want to be considered should be crossed out. Good luck!

**University Honor Pledge:** *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.*

**Signature:** ________________________________

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|---|---|---|---|
| Problem 1: | ________/4 | Problem 2: | ________/12 |
| Problem 3: | ________/4 | Problem 4: | ________/12 |
| Problem 5: | ________/8 | Problem 6: | ________/8 |
| Problem 7: | ________/8 | Problem 8: | ________/8 |
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Total Score: ________/100 Grade: ________

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(1) [4] Give the interval of definition for the solution of the initial-value problem

$$u''' - \frac{\sin(3t)}{4+t}\,u'' + \frac{5+t}{5-t}\,u = \frac{e^{-t}}{7+t}\,, \qquad u(2) = u'(2) = u''(2) = -3\,.$$

(2) [12] Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (listed with their multiplicities) are $-2+i5$, $-2+i5$, $-2-i5$, $-2-i5$, $i7$, $-i7$, $-3$, $-3$, $4$, $0$, $0$, $0$.

(a) [2] Give the order of L.

(b) [10] Give a real general solution of the homogeneous equation $\mathrm{L}y = 0$.

(3) [4] Suppose that $V_1(t)$, $V_2(t)$, and $V_3(t)$ are solutions of the differential equation

$$v''' - 2v'' - \cos(4t)v' + (1+t^2)v = 0\,,$$

Suppose we know that $\mathrm{Wr}[V_1, V_2, V_3](0) = 3$. Find $\mathrm{Wr}[V_1, V_2, V_3](t)$.

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(4) [12] The functions $\cos(4t)$ and $\sin(4t)$ are a fundamental set of solutions to $\ddot{x}+16x = 0$.

(a) [9] Solve the general initial-value problem

$$\ddot{x} + 16x = 0\,, \qquad x(0) = x_0\,, \quad \dot{x}(0) = x_1\,.$$

(b) [3] Find the associated natural fundamental set of solutions to $\ddot{x} + 16x = 0$.

(5) [8] What answer will be produced by the following Matlab commands?

```
>> ode = 'D2y – 6*Dy + 18*y = 18*exp(3*t)';
>> dsolve(ode, 't')

ans =
```

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(6) [8] Compute the Green function $g(t)$ associated with the differential operator

$$\mathrm{D}^2 + 6\mathrm{D} + 10\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

(7) [8] Solve the initial-value problem

$$q'' + 6q' + 10q = \frac{4e^{-3t}}{\cos(t)}\,, \qquad q(0) = q'(0) = 0\,.$$

(8) [8] Find a particular solution $u_P(t)$ of the equation $u'' - u = 8e^t$.

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(9) [10] The functions $1 + 2t$ and $e^{2t}$ are solutions of the homogeneous equation

$$t\,x'' - (1 + 2t)x' + 2x = 0 \qquad \text{over } t > 0\,.$$

(You do not have to check that this is true!)

(a) [3] Show that these functions are linearly independent.

(b) [7] Give a general solution of the nonhomogeneous equation

$$t\,y'' - (1 + 2t)y' + 2y = \frac{8t^2}{1 + 2t} \qquad \text{over } t > 0\,.$$

(10) [8] Give a real general solution of the equation

$$\mathrm{D}^2 v - 5\mathrm{D}v + 4v = 10\cos(3t)\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

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(11) [10] The vertical displacement of a spring-mass system is governed by the equation

$$\ddot{h} + 10\dot{h} + 169h = a\cos(\omega t - \phi)\,,$$

where $a > 0$, $\omega > 0$, and $0 \leq \phi < 2\pi$.

(a) [2] Give the natural frequency and period of the system.
(b) [4] Show the system is under damped and give its damped frequency and period.
(c) [4] Find the steady state of the system and give its phasor.

(12) [8] When a 10 gram mass is hung vertically from a spring, at rest it stretches the spring 20 cm. (Gravitational acceleration is $g = 980$ cm/sec$^2$.) A dashpot imparts a damping force of 280 dynes (1 dyne = 1 gram cm/sec$^2$) when the speed of the mass is 2 cm/sec. Assume that the spring force is proportional to displacement, that the damping force is proportional to velocity, and that there are no other forces. At $t = 0$ the mass is displaced 5 cm below its rest position and is released with a upward velocity of 4 cm/sec.

(a) [6] Formulate an initial-value problem that governs the motion of the mass for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)
(b) [2] Is this system undamped, under damped, critically damped, or over damped? (Give your reasoning!)