**Second In-Class Exam**
**Math 246, Professor David Levermore**
**Thursday, 15 March 2018**

**Your Name:** ______________________________

**UMD SID:** ______________________________

**Discussion Instructor (circle one):** Kilian Cooley Corry Bedwell Thien Ngo
**Discussion Time (circle one):** 8:00 9:00 10:00 11:00

**No books, notes, calculators, or any electronic devices.** If you need more space to answer a problem then use the back of one of these pages. Clearly indicate where your answer to each part of every problem is located. Any work that you do not want to be considered should be crossed out. **Your reasoning must be given for full credit.** Good luck!

**University Honor Pledge:** *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.* ______________________________

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**Signature:** ______________________________

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| Problem 1: | ________/4 | Problem 2: | ________/12 |
| Problem 3: | ________/4 | Problem 4: | ________/12 |
| Problem 5: | ________/8 | Problem 6: | ________/8 |
| Problem 7: | ________/8 | Problem 8: | ________/8 |
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Total Score: ________/100 Grade: ________

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(1) [4] Give the interval of definition for the solution of the initial-value problem

$$k''' - \frac{\cos(2t)}{6+t}\, k'' + \frac{e^{3t}}{2-t}\, k = \frac{6+t}{6-t}\,, \qquad k(-2) = k'(-2) = k''(-2) = 4\,.$$

(2) [12] The functions $e^{3t}$ and $e^{-3t}$ are a fundamental set of solutions to $u'' - 9u = 0$.

(a) [8] Solve the general initial-value problem

$$u'' - 9u = 0\,, \qquad u(0) = u_0\,, \quad \dot{u}(0) = u_1\,.$$

(b) [4] Find the associated natural fundamental set of solutions to $u'' - 9u = 0$.

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(3) [4] Suppose that $Z_1(t)$, $Z_2(t)$, and $Z_3(t)$ are solutions of the differential equation

$$z''' - 3z'' + (1 + t^2)z' + \sin(3t)z = 0\,,$$

Suppose we know that $\mathrm{Wr}[Z_1, Z_2, Z_3](0) = 3$. Find $\mathrm{Wr}[Z_1, Z_2, Z_3](t)$.

(4) [12] Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (listed with their multiplicities) are $-3 + i4$, $-3 + i4$, $-3 - i4$, $-3 - i4$, $-2$, $-2$, $-2$, $0$, $0$.

(a) [2] Give the order of L.

(b) [7] Give a real general solution of the homogeneous equation $\mathrm{L}v = 0$.

(c) [3] Give the degree, characteristic, and multiplicity for the forcing of the nonhomogeneous equation $\mathrm{L}w = t^2e^{-2t}$.

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(5) [8] What answer will be produced by the following Matlab commands?

```
>> ode = 'D2y - 6*Dy + 34*y = 5*exp(3*t)';
>> dsolve(ode, 't')

ans =
```

(6) [8] Find a particular solution $w_P(t)$ of the equation $w'' - w = 8t\,e^t$.

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(7) [8] Compute the Green function $g(t)$ associated with the differential operator

$$\mathrm{D}^2 + 4\mathrm{D} + 13\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

(8) [8] Solve the initial-value problem

$$x'' + 4x' + 13x = \frac{9e^{-2t}}{\sin(3t)}\,, \qquad x(\tfrac{\pi}{6}) = x'(\tfrac{\pi}{6}) = 0\,.$$

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(9) [10] The functions $1+3t$ and $e^{3t}$ are solutions of the homogeneous equation

$$t\,p'' - (1+3t)p' + 3p = 0 \qquad \text{over } t > 0\,.$$

(You do not have to check that this is true!)

(a) [3] Show that these functions are linearly independent.

(b) [7] Give a general solution of the nonhomogeneous equation

$$t\,q'' - (1+3t)q' + 3q = \frac{27t^2}{1+3t} \qquad \text{over } t > 0\,.$$

(10) [8] Give a real general solution of the equation

$$\mathrm{D}^2 v - 5\mathrm{D}v + 4v = 10\cos(3t)\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

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(11) [10] The vertical displacement of a spring-mass system is governed by the equation

$$\ddot{h} + 14\dot{h} + 625h = a\cos(\omega t - \phi)\,,$$

where $a > 0$, $\omega > 0$, and $0 \le \phi < 2\pi$. Assume CGS units.

(a) [2] Give the natural frequency and period of the system.

(b) [4] Show the system is under damped and give its damped frequency and period.

(c) [4] Give the steady state solution in its phasor representation $\mathrm{Re}(\gamma\, e^{i\omega t})$.

(12) [8] When a 10 gram mass is hung vertically from a spring, at rest it stretches the spring 9.8 cm. (Gravitational acceleration is $g = 980$ cm/sec$^2$.) A dashpot imparts a damping force of 400 dynes (1 dyne = 1 gram cm/sec$^2$) when the speed of the mass is 2 cm/sec. Assume that the spring force is proportional to displacement, that the damping force is proportional to velocity, and that there are no other forces. At $t = 0$ the mass is displaced 4 cm above its rest position and is released with a downward velocity of 3 cm/sec.

(a) [6] Give an initial-value problem that governs the displacement $h(t)$ for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)

(b) [2] Is this system undamped, under damped, critically damped, or over damped? (Give your reasoning!)