First In-Class Exam
Math 246, Professor David Levermore
Thursday, 15 February 2018

Your Name: ______________________________

UMD SID: ______________________________

Discussion Instructor (circle one): Kilian Cooley Corry Bedwell Thien Ngo
Discussion Time (circle one): 8:00 9:00 10:00 11:00

No books, notes, calculators, or any electronic devices. If you need more space to answer a problem then use the back of one of these pages. Clearly indicate where your answer to each part of every problem is located. **Your reasoning must be given for full credit.** Any work that you do not want to be considered should be crossed out. Good luck!

University Honor Pledge: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.* ______________________________

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Signature: ______________________________

Problem 1:	____/6	Problem 2:	____/22
Problem 3:	____/12	Problem 4:	____/12
Problem 5:	____/6	Problem 6:	____/6
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Problem 9:	____/20	Total Score:	____/100
		Grade:	____

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(1) [6] In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population such that it would triple every four weeks. There are 200,000 mosquitoes in the area when a flock of birds arrives that eats 50,000 mosquitoes per week.

(a) [4] Give an initial-value problem that governs $M(t)$, the number of mosquitoes in the area after the flock of birds arrives. (Do not solve the initial-value problem!)

(b) [2] Is the flock of birds large enough to control the mosquitoes?

(2) [22] Find an explicit solution for each of the following initial-value problems and give its interval of definition.

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}t} + \dfrac{4ty}{1+t^2} = \dfrac{2}{(1+t^2)^2}\,, \qquad y(0) = 3.$

(b) $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{z^2-9}{6}\,e^t\,, \qquad z(0) = 2.$

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(3) [12] Consider the differential equation $\dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{u^2(4-u)(6-u)}{(1+u^2)(2-u)^2}$.

(a) [7] Sketch its phase-line portrait over the interval $-2 \leq u \leq 8$. Identify points where it has no solution. Identify its stationary points and classify each as being either stable, unstable, or semistable.

(b) [5] For each stationary point identify the set of initial-values $u(0)$ such that the solution $u(t)$ converges to that stationary point as $t \to -\infty$.

(4) [12] Consider the following MATLAB function M-file.

```
function [t,x] = solveit(tI, xI, tF, n)

t = zeros(n + 1, 1); x = zeros(n + 1, 1);
t(1) = tI; x(1) = xI; h = (tF - tI)/n; hhalf = h/2;
for k = 1:n
thalf = t(k) + hhalf; t(k + 1) = t(k) + h;
fnow = (x(k))^2 + exp(t(k)*x(k)); xhalf = x(k) + hhalf*fnow;
fhalf = (xhalf)^2 + exp(thalf*xhalf); x(k + 1) = x(k) + h*fhalf;
end
```

Suppose the input values are tI = 2, xI = 0, tF = 10, and n = 40.

(a) [4] What initial-value problem is being approximated numerically?

(b) [2] What is the numerical method being used?

(c) [2] What is the step size?

(d) [4] What will be the output values of t(2) and x(2)?

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(5) [6] Give the interval of definition for the solution of the initial-value problem

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\cos(t)}{t^2 - 25}\, v = \frac{t}{\sin(t)}\,, \qquad v(4) = -9\,.$$

(You do not have to solve this equation to answer this question!)

(6) [6] Sketch the graph that would be produced by the following Matlab commands.

```
[X, Y] = meshgrid(−3:0.1:3,−3:0.1:3)
contour(X, Y, Y − X.ˆ2, [−2, 0, 2])
axis square
```

(7) [8] Suppose you have used a numerical method to approximate the solution of an initial-value problem over the time interval $[1, 5]$ with 1000 uniform time steps. About how many uniform time steps do you need to reduce the global error of your approximation by a factor of $\frac{1}{256}$ if the method you had used was each of the following? (Notice that $256 = 4^4$.)

(a) Runge-Kutta method

(b) Runge-midpoint method

(c) Runge-trapezoidal method

(d) Euler method

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(8) [8] A tank has a square base with 3 meter edges, a height of 2 meters, and an open top. It is initially empty when water begins to fill it at a rate of 6 liters per minute. The water also drains from the tank through a hole in its bottom at a rate of $4\sqrt{h}$ liters per minute where $h(t)$ is the height of the water in the tank in meters.

(a) [6] Give an initial-value problem that governs $h(t)$. (Recall 1 $\mathrm{m}^3 = 1000$ lit.) (Do not solve the initial-value problem!)

(b) [2] Does the tank overflow?

(9) [20] For each of the following differential forms determine if it is exact or not. If it is exact then give an implicit general solution. Otherwise find an integrating factor. (You do not need to find a general solution in the last case.)

(a) $(4xy + 3y^3)\,\mathrm{d}x + (x^2 + 3xy^2)\,\mathrm{d}y = 0\,.$

(b) $\big(e^{x+y} - \sin(x)\big)\,\mathrm{d}x + (e^{x+y} + 5y^4)\,\mathrm{d}y = 0\,.$