Math 403, Jeffrey Adams

Test I, February 12, 2010 Open Book

1. Let $G = \{1, 2, \dots, 10\}$, the non-zero integers $\pmod{11}$, considered as a group under multiplication.

   (a) Compute the order of each element of G.

   (b) Show that G is cyclic.

   (c) Find a subgroup H of G of order 2.

   (d) Find another subgroup K of G of order 5.

   (e) Show that every element g of G can be written uniquely in the form $g = xy$ with $x \in H$ and $y \in K$.

2. The symmetry group C of the cube has order 24. What are the orders of its elements? For each such order, how many elements does C have of that order? (You should thus have a list of numbers adding up to 24).

3. If G is a group, consider the map $f : G \to G$ given by $f(g) = g^{-1}$. Show that f is a homomorphism if and only if G is abelian. Assuming G is abelian, show that f is an isomorphism.

4. Let $G = GL(2, \mathbb{R})$, the two-by-two matrices of non-zero determinant.

   (a) Show that $f(g) = \text{determinant}(g)$ is a homomorphism from G to the non-zero real numbers (with multiplication).

   (b) Let $H = SL(2, \mathbb{R})$, the matrices of determinant 1. Show that H is a normal subgroup of G.