## TEST I MATH 403, J. ADAMS Open Book, not open note

Each question 25 points.

Question 1.

(a) Find the greatest common divisor (99, 57), and write it in the form 99m+57n.

(b) If a|m, b|m and (a, b) = 1, show that ab|m. *Hint*: This can be proved in a way very similar to the proof of Theorem 1.5.5 in the book.

Question 2.

(a) Find the orders of all elements of  $U_1 8$ .

(b) Give an explicit isomorphism  $U_{18} \simeq \mathbb{Z}/6\mathbb{Z} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}.$ 

(c) Show that  $U_{12}$  is not a cyclic group.

Question 3.

(a) Suppose G, G' are groups, and  $\phi$  is a homomorphism of G onto G'. Prove that if G is abelian then G' is abelian.

(b) Is the converse true: i.e. if G' is abelian, then G is abelian? Prove or give a counterexample.

Question 4.

Let S be a set, and consider the group A(S) of 1–1 mappings of S to itself. For  $s \in S$ , let  $H_s = \{f \in A(S) | f(s) = s\}$ .

(a) Show that  $H_s$  is a subgroup of A(S).

(b) Suppose  $f(s) = t \neq s$ . What is  $fH_sf^{-1}$ ? Is  $H_s$  a normal subgroup of A(S)? Briely state why or why not.