**Math 403, Jeffrey Adams**

Test II, April 30, 2010 Take Home, Due Monday May 3

Honor system: open book, but no online or other extensive research

Throughout $R$ is a ring.

1. Suppose $R$ has no zero-divisors, and $a \in R$ ($a \neq 0$) satisfies $a^2 = a$. Show that $a$ is a unity for $R$.

2. Suppose $R$ is commutative with prime characteristic $p$.

   (a) Show that for all $a, b \in R$, $(a+b)^p = a^p + b^p$.

   (b) Show that the map $f(a) = a^p$ is a ring homomorphism from $R$ to $R$.

3. Suppose $R$ is commutative with unity. Let $S = \{r \in R \mid r \text{ is } \textit{not} \text{ a unit}\}$. If $S$ is an ideal, show that it is (a) a maximal ideal in $R$, and (b) the unique maximal ideal.

4. Suppose $R, S$ are commutative with unities. Let $f$ be a homomorphism from $R$ onto $S$. Suppose $I$ is an ideal in $S$, and let $J = \{r \in R \mid f(r) \in I\}$.

   (a) Show that $J$ is an ideal in $R$.

   (b) If $I$ is prime show that $J$ is prime.

   (c) If $I$ is maximal show that $J$ is maximal.

5. Suppose $R$ is commutative and $I$ is a prime ideal of $R$. Show that (a) $I[x]$ is an ideal in $R[x]$ and (b) $I[x]$ is a prime ideal.

6. For $p$ a prime determine the number of irreducible polynomials over $\mathbb{Z}_p$ of degree 2.